Product Rationalization and Trade Liberalization

By

Hong Ma

University of California, Davis

June 2008

[Preliminary]

Abstract: The Eastman-Stykolt hypothesis (Eastman and Stykolt 1967) suggests that in a small open economy after trade liberalization, firms expand in scale (the intensive margin) but contract in scope (the extensive margin). This hypothesis has been confirmed in firms' rationalization in Canada after the 1989 Canada-U.S. FTA. This paper provides a theoretical justification for this result using a monopolistic competition model with multi-product firms. We argue that the firm adjusts its scope of products in response to trade shocks. With identical costs across firms, trade liberalization leads to rationalization in firms' product range but expansion in export, while firm-level selection and scale are not changed. With heterogeneous costs, exporting firms expand their export range but contract on their total number of products in response to declining trade costs. Fewer firms survive in equilibrium, while a higher proportion serve the export market. Exporting firms' average scale of production increases.

1. INTRODUCTION

The Eastman-Stykolt hypothesis (Eastman and Stykolt 1967, Muller and Rawana 1990) suggests that the presence of tariff protections leads to highly concentrated industries for a small economy like Canada, in which firms proliferate in product lines but do not exhaust scale economies. This hypothesis predicts that trade liberalization, in form of a reduction in trade costs, would encourage firms to expand product length and to rationalize the number of products they provide. Hence moving up on the average cost curve, the average productivity of each industry is increased and consumer welfare is improved.

This prediction, along with the computable general equilibrium model developed in Harris (1984) and Cox and Harris (1985, 1986), argues that reduction in tariffs could generate greater gains from trade for Canada than conventionally thought. This argument has led to much empirical work studying the impact of Canada-U.S. Free Trade Agreement (CAUSFTA) in 1989 and the following North American Free Trade Agreement (NAFTA) in 1994, on production and welfare in Canada. Baldwin and Gorecki (1983), for example, document that average Canadian plant size increased by 33 percent over the 1970s period of trade liberalization. And rationalization was found particularly strong in industries with initially high tariffs and relatively large minimum efficient scales of production. Head and Ries (1999) also find a dramatic rationalization effect after the Canada-U.S. FTA: there were fewer firms producing at larger scale. Furthermore, their detailed study, on a sample of 230 Canadian manufacturing industries over 1988-1994, finds that a reduction in Canadian tariffs reduces the average size of Canadian plants and also the number of plants, while a reduction in U.S. tariffs has the opposite effect on scale but no effect on firms' entry. Baldwin, Beckstead and Caves (2001) examine both firm and plant level specialization and find evidence of commodity specialization at the plant level, in particular for the exporting firms. Trefler (2004) provides evidence that firm selection (exit of low-productivity plants) instead of larger production scale, is one major source of productivity gain after CAUSFTA was established.

These empirical findings lead to the theoretical question of how to best model scale economies and multiple products per firm. Eastman and Stykolt themselves suggest domestic firms collude by setting their prices at the importer's CIF price plus the tariff. This tariff-limit pricing setup disregards the consideration of exports and is not formalized satisfactorily (Muller, 1982). On the other hand, conventional oligopolistic and monopolistic models as surveyed in Head and Ries (1999) provide different (if not controversial) implications and do not recognize the fact that firms actually produce multiple products.[1] Therefore, the impact of trade liberalization on product diversification is barely addressed in theory. Baldwin and Gu (2005) fill this gap by introducing multiproduct firms into the Melitz-Ottaviano framework (2008). In their model, firms have heterogeneous marginal costs and produce multiple varieties, while consumers are assumed to have quadratic utility function. They predict bilateral trade liberalization reduces the number of products supplied by plants and increases the production-run length of exporters. However, the impact on plant size could be ambiguous.

This paper provides an alternative theoretical justification using a monopolistic competition model with CES preferences. Similar to Baldwin and Gu, we allow firms to optimize on product scope: each firm produces multiple products and compete in a differentiated good market. In fact, Eastman and Stykolt (1967) do realize that adding production capacity within a firm is an alternative form of entry.

The multiproduct firm model has been supported by recent empirical findings in the international trade literature. As documented in Bernard, Redding and Schott (2006a), 41% of U.S. manufacturing firms produce in multiple 5-digit SIC industries, accounting for 91% of total sales. Firms with larger sales usually manage more product lines than relatively smaller firms, which contributes substantially to the sales variation across firms (Nocke and Yeaple, 2006). The concentration of sales in very large, multiproduct firms is even more apparent when we look at their exports sales. Bernard, Jensen and Schott (2007) show that

[1] For oligopolistic competition, see Venables (1985) and Horstman and Markusen (1986), etc.; for monopolistic competition, see Helpman and Krugman (1985).

the top 1% of U.S. trading firms account for over 80% of total trade in 2000. Over 10% of exporters and 20% of importers are trading 10 or more harmonized system (HS) products, and these firms account for about 90% of export and import value. The concentration is also confirmed in Arkolakis and Muendler (2008) by looking at a large sample of Brazilian exporting firms. Therefore, much of the expansion and contraction of firms should actually be accounted for by the extensive margin of adding and dropping products, rather than the intensive margin of changing output of existing products.

Our model is an extension to the multiproduct model developed in Allanson and Montagna (2005). Deviated from their analysis on a closed economy and identical firms, we focus on the impact of tariff reduction on firm behavior, and in addition, we also introduce firm heterogeneity in costs, which following the seminal work of Melitz (2003). The existence of heterogeneity across firms implies interesting selection among firms. Furthermore, because of the fixed beachhead costs of exporting, there are also selections of products within each exporting firm, implying interesting adjustments of extensive margin within a firm.

For simplicity, we drop the small country assumption which is essential in Eastman and Stykolt's original argument, but assume identical countries. Nonetheless, our simple specification generates rich and interesting results. First, with identical costs, selection only occurs within each firm. A reduction in bilateral tariffs leads to rationalization in firms' product range for the domestic market but also an expansion in their product range for export. However, adjustment in firm scale is absent, which is analoguous to the conventional monopolistic competition model as in Krugman (1980), where firms enter and exit without changes in production scale. Second, with heterogeneous costs, more productive firms not only have longer product runs, but also produce a greater range of varieties. Given their endowed productivity level, firms self-select into three cohorts: the non-producer, the local producer and the exporter. Only the most productive firms will find it profitable to export abroad, but they will only export a proportion of their products. In response to declining trade cost, exporting firms expand their export range but contract on total number of products they develop. The least productive firms do not export and exit when trade costs drop. Fewer firms survive in equilibrium, while a higher fraction of them serve the export market.

The rest of the paper is organized as follows. The basic model is set up in Section 2. Section 3 shows the case of identical costs across firms. We then introduce heterogeneous productivity in Section 4 and discuss the impact of trade liberalization on heterogeneous firms. Section 5 concludes.

2. PREFERENCES, DEMAND AND FIRMS

There are two countries, home (H) and the rest of the world (F). There are L^c consumers (workers) in economy $c = H, F$. Assuming identical countries, then the wage in both countries can be normalized to one. We focus on the home country. Each consumer is endowed with one unit of labor and has a CES utility function over a continuum of products:

$$U = \left(\int_{i=0}^{N^H} q(i)^{(\eta-1)/\eta} di + \int_{i'=0}^{N^F} q(i')^{(\eta-1)/\eta} di' \right)^{\eta/(\eta-1)}, \eta > 1. \tag{1}$$

where N^c denotes the number of firms sourced from country $c = H, F$. Each firm i supplies a continuum of n_i^c varieties with the aggregate quantity $q(i)$ expressed as:

$$q(i) = \left(\int_{j=0}^{n_i} q_i(j)^{(\sigma-1)/\sigma} di \right)^{\sigma/(\sigma-1)}, \sigma > 1. \tag{2}$$

where $\sigma > \eta$. This two-tier CES utility function captures the idea that varieties are more substitutable within a firm than between firms. Because of the homotheticity of the preference, we solve the utility maximization problem by two-stage budgeting, which gives the demand for each variety:

$$q_i(j) = Y P^{\eta-1} P(i)^{\sigma-\eta} p_i(j)^{-\sigma} \tag{3}$$

where Y denotes the total expenditure on the differentiated products. The aggregate price index P is defined as $P^{1-\eta} = \int_{i=0}^{N^H} P(i)^{1-\eta} di + \int_{i'=0}^{N^F} P(i')^{1-\eta} di'$, and the firm level price index $P(i)$ is defined as $P(i)^{1-\sigma} = \int_{j=0}^{n_i} p_i(j)^{1-\sigma} dj$.

This multiproduct framework has been widely utilitized in the IO literature. With few exceptions, most of that work explores oligopolistic competition among symmetric firms with or without free entry (see, for example, Raubitschek 1987, Ju 2003)[2]. Taking into account the marginal cost and quality differences, Grossmann (2007) shows that firms with higher quality-cost margins are larger and more diversified. Allanson and Montagna (2005) apply a monopolistic competition model with symmetric multiple-prduct firms to explain the industry shakeout process. All those models are closed-economy analysis without international trade flows. In the international trade literature, the earliest work on multiproduct firms is by Helpman (1985). Bernard, Redding and Schott (2006b) propose a multi-product-multi-industry model under asymmetric productivity within and between firms. In their set-up, each firm is randomly endowed with a firm-level ability and choose to enter a continuum of monopolistic competitive markets where it has a specific "expertise" in production. Distinct from their "interlacing" market structure where each firm enters different product markets, our model assumes each firm produce multiple varieties competing with rivals in the same market. Firms could be different in productivity, but varieties within a firm have identical marginal costs. Agur (2007) uses a similar model to analyze the welfare effect of firm level variety growth. The model closest to this paper is Arkolakis and Muendler (2007), which develop a multiproduct model with two-tier CES preference and diseconomies of scope. Using their firm-level data on Brazilian exporting firms, they confirm that many firms produce more than one product and the distributions of firms' sales and product scope approximately follow Pareto, which is also consistent with this paper.

To be concrete, we model the total production cost function of firm i as:

$$C(i) = \alpha_d + \int_{j=0}^{n_i} (f_d + c(i)q_i(j))dj + I \cdot \left(\alpha_x + \int_{j=0}^{n_{xi}} (f_x + \tau c(i) q_i^x(j))dj \right)$$

where I is an indicator variable: $I = 1$ if firm i actually exports and $= 0$ otherwise. $c(i)$ denotes the marginal cost of production which applies to all varieties of firm i. $q_i(j)$ denotes the quantity sold in domestic market for variety j of firm i. With a superscript x, $q_i^x(j)$ denotes the quantity of the variety sold abroad. f_d and f_x represent the varity-level fixed

[2] Anderson and de Palma (1992) propose a nested multinomial logit model to analyse the Bertrand Competition between multiproduct firms, which is out of this paper's scope. Neither will we consider market segmentation or collusive activities between multiproduct firms.

costs incured when a firm develop a variety or export a variety. α_d and a_x, on the other hand, represent the firm-level benchhead costs incured when a firm setups product facilities or exports to a foreign markets.

The timing of the model is: first, firms pay a sunk cost f_e to enter the market and acquire the knowledge of their own productivity level; second, firms decide whether to stay in the domestic market and whether to export[3]; finally, active firms decide on the prices and number of varieties they will supply to either market. In addition to the sunk cost of entry, each firm needs to invest α_d as the fixed cost over all varieties for sales in domestic market, and to supply each additional variety to the domestic market, it needs to invest f_d. Furthermore, the fixed cost of export has been recognized in the empirical works in international trade (see, for example, Roberts and Tybout 1997, Bernard and Wagner 2001). In this model, exporting an additional variety abroad incurs f_x and managing all exporting varieties incurs a beachhead fixed exporting costs α_x.

As standard in the monopolistic competition model, firms take as exogenous the general market environment $YP^{\eta-1}$. Furthermore each firm produces a continuum of varieties such that the firm-level price index $P(i)$ is also taken as given by each variety manager. This way we exclude the possible cooperation among product lines within a firm and therefore the profit maximization gives a constant markup[4] which only depends on the elasticity of substitution σ. Assuming iceberg trade costs, the pricing rule for domestic and imported varieties can be expressed as:

$$p_i = \frac{\sigma}{\sigma - 1} c(i) \text{ and } p^F_{i'} = \frac{\sigma}{\sigma - 1} \tau c(i') \tag{4}$$

where τ denotes the *ad valorem* trade costs (including tariff and transportation costs) which

[3]Note in the symmetric case developed in Section 3, ther is no sunk cost incurred for simplicity, each firm, by free entry condition, still generate zero economic profit. Nonetheless, the sunk cost will play important role in the case with heterogeneous firms.

[4]If a firm does take into account of the impact of the price change in one variety on other varieties within the firm boundary, but in any case neglects the aggregate price index, then the markup will be solely determined the across-firm substitutability η, instead of σ appearing in (4). Most qualitative results are unaffected by this change.

is bilaterally symmetric. Multiproduct firms not only make price decisions, but also need to choose the optimal product scope — this will be discussed later in following sections.

The home country H is assumed to be identical to the foreign F, thus the importing behavior of firms from F mirrors the exporting behavior of home firms. This concludes the introduction of the basic model set-up. We then consider two cases: in section 3, we discuss the case where all firms have symmetric marginal costs, while in section 4, we extend the model to incorporate the Melitz style heterogenity of productivity across firms.

3. FIRMS WITH IDENTICAL COSTS IN AN OPEN ECONOMY

In our basic model, we follow Krugman (1980) in assuming all firms share the same production technology and therefore have identical marginal costs. Without loss of generosity we assume ϕ units of labor produce one unit of output. Since $w = 1$, we have $c(i) = \phi, \forall i$. Using the demand function (3) and the pricing rule (4), we could get the quantity, revenue, and operating profit for selling *one* variety to the domestic market[5]:

$$q_d = A^H n_d^{(\sigma-\eta)/(1-\sigma)} (\frac{\phi}{\rho})^{-\eta}, \quad r_d = A^H n_d^{(\sigma-\eta)/(1-\sigma)} (\frac{\phi}{\rho})^{1-\eta}, \ \pi_d = \frac{1}{\sigma} r_d - f_d$$

where $\rho = \frac{\sigma-1}{\sigma}$. $A^H = Y^H P^{\eta-1}$ represents the aggregate market condition, which will be taken as given by firms. n_d denotes the number of varieties produced by a single firm. By symmetry, the aggregate price index is $P^{1-\eta} = N^H n_d^{(\eta-1)/(\sigma-1)} (\frac{\phi}{\rho})^{1-\eta} + N^F n_x^{(\eta-1)/(\sigma-1)} (\frac{\tau\phi}{\rho})^{1-\eta}$.

Similarly, the quantity, revenue and operating profit for exporting *one* variety to the foreign market are:

$$q_x = A^F n_x^{(\sigma-\eta)/(1-\sigma)} (\frac{\tau\phi}{\rho})^{-\eta}, \quad r_x = A^F n_x^{(\sigma-\eta)/(1-\sigma)} (\frac{\tau\phi}{\rho})^{1-\eta}, \ \pi_x = \frac{1}{\sigma} r_x - f_x$$

The total operating profit for domestic sales is $\Pi(n_d) = n_d(\frac{1}{\sigma} r_d - f_d) - \alpha_d$, and the total operating profit for export sales is $\Pi(n_x) = n_x(\frac{1}{\sigma} r_x - f_x) - \alpha_x$.

Maximizing profit with respect to the number of products gives the optimal scope:

$$n_d^{(\sigma-\eta)/(\sigma-1)} = \frac{\eta-1}{\sigma(\sigma-1)} \frac{A^H}{f_d} (\frac{\phi}{\rho})^{1-\eta} \tag{5}$$

[5] Note that since all firms are identical, we suppress the firm index for simplicity.

$$n_x^{(\sigma-\eta)/(\sigma-1)} = \frac{\eta-1}{\sigma(\sigma-1)}\frac{A^F}{f_x}(\frac{\tau\phi}{\rho})^{1-\eta} \tag{6}$$

Because the two countries are identical, we must have $A^H = A^F$. Further, since all firms are identical, all domestic sellers are also exporters, and $N^H = N^F$. However, the product range for domestic market does not necessarily equal the product range for export. Actually, taking ratio of (5) and (6), we have:

$$\frac{n_d}{n_x} = \left(\frac{\tau^{\eta-1} f_x}{f_d}\right)^{(\sigma-1)/(\sigma-\eta)}. \tag{7}$$

Higher trade costs, either the iceberg transportation costs or the beachhead fixed investment, discourage firms to expand the full range of varieties for export. The *regularity condition* $\tau^{\eta-1} > \frac{f_d}{f_x}$ is needed to ensure firms do not innovate new varieties purely for export.

In equlibrium, free entry ensures that no firm get positive operating profit. Using (5) and (6), we get:

$$\Pi(n_d, n_x) = \Pi(n_d) + \Pi(n_x) = \frac{\sigma-\eta}{\eta-1}(n_d f_d + n_x f_x) - \alpha_d - \alpha_x = 0 \tag{8}$$

Using the variety ratio (7), the number of varieties can be solved as a function of marginal and fixed costs:

$$n_d = \frac{\eta-1}{\sigma-\eta}\frac{\alpha_d+\alpha_x}{f_d}\theta, \quad \text{and} \quad n_x = \frac{\eta-1}{\sigma-\eta}\frac{\alpha_d+\alpha_x}{f_x}(1-\theta). \tag{9}$$

where $\theta = \left(1 + \left(\frac{f_x}{f_d}\right)^{-(\eta-1)/(\sigma-\eta)} (\tau)^{-(\eta-1)(\sigma-1)/(\sigma-\eta)}\right)^{-1}$.

Since $w = 1$, total expenditure equals labor income as $Y^H = L^H$. Using (5), and notice that $A^H = Y^H P^{\eta-1}$, and $P^{1-\eta} = N^H \left(n_d^{(\eta-1)/(\sigma-1)}(\frac{\phi}{\rho})^{1-\eta} + n_x^{(\eta-1)/(\sigma-1)}(\frac{\tau\phi}{\rho})^{1-\eta}\right)$ (by symmetry $N^H = N^F$ and use (9)), we get the number of incumbent multi-product firms:

$$N^H = N^F = \frac{\sigma-\eta}{\sigma(\sigma-1)}\frac{L^H}{\alpha_d+\alpha_x} \tag{10}$$

Intuitively, higher beachhead costs of selling in either market make entry less likely such that incumbent firms tend to expand in their product range. From (7), the expansions in domestic and export product range are proportional, given that τ, f_d and f_x fixed.

Trade liberalization, in the form of reductions in iceberg transport costs or in product-level fixed cost f_x, raises the profitability of export and encourages firms to export more varieties. Interestingly, more imported varieties, on the other hand, drives up the real cost of factor employment and therefore force firms to trim their domestic product lines.

The population of incumbent firms is larger, the larger is the country size, the smaller is the sunk cost of entry. Lower substitutability among varieties provided by different firms alleviates competition and therefore has more firms survive in equilibrium. This is also the conclusion reached in Allanson and Montagna (2005). Trade liberalization, however, does not have impact on the number of incumbent firms.

Finally, due to the constant elasticity of demand (which is a convenient feature of the Dixit-Stiglitz (1976) style monopolistic competition model), trade liberalization in terms of reducing iceberg trade costs does not have impact on firms' scale. This could be easily checked using the demand function (3), which gives[6]:

$$q_d = \frac{(\sigma-1)^2}{(\eta-1)}\frac{f_d}{\phi}, \quad \text{and} \quad \tau q_x = \frac{(\sigma-1)^2}{(\eta-1)}\frac{f_x}{\phi},$$

and the total output shipped by one firm is $n_d q_d + n_x \tau q_x = \frac{(\sigma-1)^2}{(\sigma-\eta)}\frac{\alpha_d+\alpha_x}{\phi}$. A decrease in fixed cost f_d and f_x will increase the shipment q_d and τq_x, but the increments are exactly offset by the decrease in the number of varieties a firm develops for each market. There is not scale adjustmet for each variety in response to changes in trade costs, as in Krugman (1980).

Comparative statistics analysis leads to the following result:

Proposition 1 *Assuming identical marginal costs and symmetric countries,*

(1). lower trade cost τ or lower relative fixed cost of export f_x/f_d leads to rationalization in firms' range of varieties (n_d), but expansion in firms' range of export varieties (n_x);

(2). Higher fixed cost of selling in either market ($\alpha_d + \alpha_x$) leads to fewer firms active in equilibrium, but each firm produce more varieties for both domestic market and abroad, so

[6] For quantity of export, note by the iceberg trade cost assumption, for q_x units of product to arrive, one need to ship τq_x units. The symmetric CES structure ensures that the shipment of foreign export τq_x does not change with τ, while the net arrival q_x decreases with τ.

the total number of varieties keeps constant;

(3). Trade costs have no impact on firm's scale.

To conclude, with identical costs, trade liberalization leads to rationalization in the range of total varieties a firm produces but expansion in the range of exporting varieties. This is the "scope effect" found in this multiproduct framework. However, firm-level selection and the "scale effect" is absent: all firms participate in export activities, and each firm ships the same quantity, with or without trade. At last, consumers certainly have welfare gain from trade, because their preferences favor varieties.

4. PRODUCTION WITH HETEROGENOUS COSTS

We now extend the basic model to allow asymmetric marginal costs across firms. Consistent with the empirical evidence, recent literature in international trade theory has emphasized the fact that an industry has a wide range of firms operating at different productivity level (Melitz 2003, Helpman, Melitz and Yeaple 2004). Here we consider a case with heterogenous firms who produce multiple product lines. We first investigate the equilibrium in autarky, and then introduce international trade.

4.1 Closed Economy Equilibrium

Firms are distinguished by their different level of marginal production costs, which is assumed to be drawn from an exogenous distribution $G(\phi)$ with support $(0, \overline{\phi})$. Firms realize their own productivity after they enter the market with sunk costs paid. According to their productivity draws, firms then self-select into different status: most productive firms would service both domestic and foreign markets, less productive firms would sell exclusively to the local market, while the least productive firms have net loss of sunk cost and exit the market immediately. As in the symmetric case, the same production technology applies to all varieties within the same firm. And due to the CES preference, a firm choose its optimal price as a constant markup over its marginal cost ϕ: $p(\phi) = \frac{\sigma}{\sigma-1}\phi$. The multiproduct firm then optimize its product scope by solving

$$\text{Max}_{n_d} \quad \Pi_d(n_d) = n(\phi)\left(\frac{1}{\sigma}Y^H P^{\eta-1} n_d^{(\sigma-\eta)/(1-\sigma)}(\frac{\phi}{\rho})^{1-\eta} - f_d\right) - \alpha_d, \;\; \rho = \frac{\sigma-1}{\sigma}, \tag{11}$$

which gives:

$$n_d(\phi) = \left(\frac{\eta-1}{\sigma(\sigma-1)}\frac{Y^H P^{\eta-1}}{f_d}(\frac{\phi}{\rho})^{1-\eta}\right)^{(\sigma-1)/(\sigma-\eta)} \tag{12}$$

Firms differ in their realized productivity and therefore differ in their profitability. The least efficient firms cannot break even and hence will exit the market immediately without any production, while the most productive firms will earn positive profits. The cutoff condition derived from equalizing the optimized profit to zero gives

$$n_d(\phi^*) = \frac{\eta-1}{\sigma-\eta}\frac{\alpha_d}{f_d}. \tag{13}$$

where ϕ^* represents the cutoff firm's marginal cost. So the cutoff firm's scope expands with the firm-level governance cost relative to the variety-level fixed cost (i.e., α_d/f_d). The cutoff marginal cost ϕ^* is endogenously determined in (12). Furthermore, from (12) for any surviving firm with marignal cost $\phi < \phi^*$, its optimal number of varieties can be expressed as:

$$\frac{n_d(\phi)}{n_d(\phi^*)} = \left(\frac{\phi}{\phi^*}\right)^{-(\eta-1)(\sigma-1)/(\sigma-\eta)}. \tag{14}$$

We see that more productive firms tend to have greater range of varieties (*the extensive margin*). The elasticity of the product range ratio with respect to their productivity ratio is $(\eta-1)(\sigma-1)/(\sigma-\eta)$. On the other hand, more productive firms also produce larger quantities in each of their products (*the intensive margin*)[7], this could be seen be comparing $q(\phi_1)$ and $q(\phi_2)$ with $\phi_1 < \phi_2$:

$$\frac{q(\phi_1)}{q(\phi_2)} = \left(\frac{\phi_2}{\phi_1}\right) > 1 \tag{15}$$

Proposition 2 *More productive firms tend to produce larger quantities in each of its varieties (the intensive margin), and they also tend to produce a greater range of varieties (the extensive margin).*

[7]If the intensive margin is defined as the sales per variety, then there is no variation in the intensive margin corresponding to the variation in productivity.

More productive firms are larger on average, not only because they produce more in a given variety, but also because they produce produce greater number of varieties. This finding is also pointed out in Bernard, Redding and Schott (2006b) (see their Proposition 1). In their paper, firms not only differ in their overall "ability", but varieties within a firm also differ in manager's expertise. Therefore, in their two-dimension heterogeneity framework, the positive correlation between the intensive margin and the extensive margin lies in the firm-level "ability", which enhances the overall productivity for each variety the firm produces. Therefore even for a variety which the firm has a poor "expertise", higher "ability" helps the firm to be profitable in producing it. In comparison, in our model, firms have only one dimension of exogenous heterogeneity — their productivity level which applies equally to the whole range of products they produce[8].

It is also interesting to look at the size distribution of firms who differ in productivity level. From (14) and (15), more productive firms tend to produce much larger quantity in total.

$$\frac{n_d(\phi_1)q(\phi_1)}{n_d(\phi_2)q(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{-1-(\eta-1)(\sigma-1)/(\sigma-\eta)}, \quad \phi_1 < \phi_2 \tag{16}$$

Furthermore, more productive firms also tend to have much greater revenue.

$$\frac{n_d(\phi_1)r(\phi_1)}{n_d(\phi_2)r(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{-(\eta-1)(\sigma-1)/(\sigma-\eta)}, \quad \phi_1 < \phi_2 \tag{17}$$

which importantly indicates that the revenue advantage of a more productive firm is derived completely from its greater variety range. Interestingly, the revenue of a single variety for a productive firm is the same as the variety revenue of a less productive firm. This is because the revenue from one variety does not only depend on the firm's productivity level (with an elasticity of $\eta - 1$), but also *negatively* depend on the number of varieties the firm introduces in total (with an elasticity of $(\sigma - \eta)/(1 - \sigma)$) — the other varieties from the same firm subtract off some demand for this variety, which in IO literature is recognized as

[8] Departing from CES preferences, Nocke and Yeaple (2006) use a partial equilibrium inverse demand system for every product produced by a firm. An increase in the number of product lines increases a firm's marginal cost. Therefore they predict a negative correlation between the number of product lines and the sales per product.

the "cannibalization effect".

With an elasticity of magnitude $(\eta - 1)(\sigma - 1)/(\sigma - \eta)$, the ability to produce a greater range of varieties magnifies the relative size of the more productive firms compared with their less efficient rivals. That is:

Proposition 3 *Given the elasticities σ and η satisfy $\sigma > \eta > 1$, the endogenous choice of product scope magnifies the size differential between heterogenous firms with multiple product lines.*

Under our multiproduct framework, the intensive margin and the extensive margin combined contribute to the magnification of firms' inequality in the size distribution. To see this, recall that we assume the elasticity of substitution within a firm is greater than the subsitution between firms (i.e., $\sigma > \eta$), so $(\eta - 1)(\sigma - 1)/(\sigma - \eta) > (\eta - 1)$. While for a typical model with single product firms and productivity heterogeneity (for example, Melitz 2003), $\frac{r(\phi_1)}{r(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{-(\eta-1)}$, therefore the elasticity of revenue differential with respect to the productivity differential is simply $(\eta - 1)$. Put it another way, a firm who is 1 percent superior to another firm will have a revenue $(\eta - 1)(\sigma - 1)/(\sigma - \eta)$ percent (instead of a $(\eta - 1)$ percent as in a single-variety firm model) higher than its rival[9]. A similar magnification in size distribution is also observed in Bernard, Redding and Schott (2006b, Corollary 1). In their paper, the magnification of size dispersion relies on the fact that more capable firm produces in more product categories in addition to the "common product set" with a less capable firm, so it enters as an additive term . While in this paper, each firm produce more than one products in the same industry and the magnification of size dispersion enters as a power term.

As will be clear soon, one non-trivial advantage of our power term magnification process is that it retains the distribution proporties of the productivity draw. In particular, following

[9] This magnification effect of productivity difference could indeed be quite significant. Consider reasonable elasticities of substitution, for example, $\eta = 5$, $\sigma = 8$, then a typical Melitz model predicts one percent increase in productivity differential leads to 4 percent increase in size differential, while our model predicts 9.3 percent!

the literature (Helpman, Melitz and Yeaple 2004, Melitz and Ottaviano 2008), we assume productivity is subject to a Pareto distribution[10].

Proposition 4 *In a closed economy, suppose firm's labor productivity $1/\phi$ is drawn from a Pareto distribution with the shape parameter γ,*

(1). the distributions of the multiproduct firm's domestic sales, number of varieties produced (the extensive margin), quantity sold for each variety (the intensive margin), and total quantity, are also subject to Pareto distribution, with the shape parameters $\gamma - (\eta - 1)(\sigma - 1)/(\sigma - \eta)$, $\gamma - (\eta - 1)(\sigma - 1)/(\sigma - \eta)$, γ, and $\gamma - 1 - (\eta - 1)(\sigma - 1)/(\sigma - \eta)$;

(2). A higher dispersion of firm productivity draws (lower γ), or a higher elasticity of substitution between firms (higer η), or a higher elasticity of substitution between varieties within a firm (higher σ), raise the dispersion of the multiproduct firm's domestic sales, number of varieties produced (the extensive margin), and total quantity.

(3). A higher dispersion of firm productivity draws (lower γ) raises the dispersion of quantity sold for each variety (the intensive margin).

Proof. see the Appendix. ■

Plugging (12) back to (11) and using (14), we can write firm's profit as:

$$\Pi(\phi) = \left[\left(\frac{\phi}{\phi^*} \right)^{-(\eta-1)(\sigma-1)/(\sigma-\eta)} - 1 \right] \alpha_d \tag{18}$$

Before entry, all potential producers are identical and don't know their true level of productivity. They pay an initial set-up investment f_e which is thereafter sunk so as to enter the market and realize their productivity draws. Firm selection occurs: the least efficient firm with very large marginal cost cannot break even and hence will leave the market without production. The borderline firm draws the cutoff cost ϕ^*, stays in the market with zero operating profits. Under free entry, the market reaches equilibrium when the *ex ante* expected operating profit is driven down to the level of sunk cost of entry. That means *the free entry condition* could be written as:

$$E(\Pi) = \int_0^{\phi^*} \left[\left(\frac{\phi}{\phi^*} \right)^{-(\eta-1)(\sigma-1)/(\sigma-\eta)} - 1 \right] \alpha_d dG(\phi) = f_e \tag{19}$$

[10] For details on the properties of a Pareto distribution, see Axtell (2001).

To solve the free entry condition explicitly, we parameterize the distribution of labor productivity ($1/\phi$) as Pareto with shape parameter γ. Therefore the marginal cost ϕ is subject to a distribution with CDF:[11]

$$G(\phi) = \left(\frac{\phi}{\overline{\phi}}\right)^{\gamma}, \phi \in (0, \overline{\phi})$$

Solving (19) gives the cutoff marginal cost ϕ^* as[12]:

$$\left(\frac{\phi^*}{\overline{\phi}}\right)^{\gamma} = \left(\frac{\gamma(\sigma-\eta)}{(\eta-1)(\sigma-1)} - 1\right)\frac{f_e}{\alpha_d} \tag{20}$$

To ensure the pareto distributions of sales and quantities converge (i.e., a finite variance)[13], we need to impose an additional regularity condition: $\gamma > 3+(\eta-1)(\sigma-1)/(\sigma-\eta)$. This simultaneously justifies the existence of solution to (20).

From (13) and (20), we have a result for the borderline firm with zero operating profit:

Proposition 5 *In a closed economy with heterogenous multiproduct firms,*

(1).the borderline firm's scope $n(\phi^*)$ increases in the beachhead fixed costs relative to the variety fixed cost (α_d/f_d);

(2).the borderline firm's cutoff marginal cost ϕ^* increases in the sunk entry cost relative to the beachhead fixed cost (f_e/α_d);

(3).firm level characteristics (i.e., scale and scope of a firm) are independent of market size.

Combining (12), (13), and (20), and using $Y^H = L^H$ (recall that $w = 1$), we can solve the number of firms active in the equilibrium:

$$N^H = \lambda\frac{L^H}{f_e} \tag{21}$$

where $\lambda = \frac{\eta-1}{\sigma}\left(\frac{\eta-1}{\sigma-\eta}\right)^{(\eta-\sigma)/(\sigma-1)}$.

[11] Helpman, Melitz and Rubinstein (2007) introduce a Pareto distribution truncated from below. So $G(\phi) = \frac{\phi^{\gamma}-\underline{\phi}^{\gamma}}{\overline{\phi}^{\gamma}-\underline{\phi}^{\gamma}}$, $\phi \in (\underline{\phi}, \overline{\phi})$. It would be interesting to model a movement of $\underline{\phi}$ as an exogenous technology advance and investigate the impact of this change.

[12] See appendix for proof.

[13] See Helpman, Melitz and Yeaple (2004), footnote 14.

It is apparent to see that the equilibrium number of firms moves proportionately with the size of the market. As in the single-product model of monopolistic competition with constant elasticity (whether with homogenous or heterogenous firms), firm-level characteristics such as the quantity, scope, and the cutoff productivity level are independent of the market size. Opening trade between two identical countries, without trade costs, is simply equivalent to doubling the country size, implying a doubling in the number of firms serving consumers in each country and thereforth gains in welfare. This result is obtained from the constant elasticity assumption.

4.2 Open Economy and The Impact of Bilateral Trade Liberalization

We have examined firm's behavior in a closed economy, which gives us some interesting implications: first, more productive firm produce in larger quantity, and also produce in greater variety range; second, the endogeneity in scope choice amplify firm's inequality; third, a larger country has more firms in equilibrium, but the average productivity level is unchanged; fourth, each firm's product range is independent of the country size. We formally introduce international trade with frictions across borders in this section.

We have already derived the pricing rules as in (4), and the scope choice for *domestic* sales is given in (12), (13), and (14). By the nature of firm heterogeneity, only a portion of all potential firms will finally produce for the domestic demand. And among them, only those who are above the threshold productivity level will also export abroad. Furthermore, for an successful exporter, it does not necessarily supply all its domestic varieties to the foreign market. In fact, as will be clear below, when a reasonable regularity condition is satisfied ($\tau^{\eta-1} > \frac{f_d}{f_x}$), an exporter will only export a fraction of its products.

Similar to what we have done in section 4.1, if a firm decide to export, it charges a constant markup for its product: $p_x(\phi) = \frac{\tau}{\rho}\phi$, with $\rho = \frac{\sigma-1}{\sigma}$. Its optimal product scope of *exports* is chosen in order to maximize its profits for selling abroad:

$$\text{Max}_{n_x} \quad \Pi(n_x(\phi)) = n_x(\phi) \left(\frac{1}{\sigma} Y^F \left(P^F\right)^{\eta-1} n_x(\phi)^{(\sigma-\eta)/(1-\sigma)} (\frac{\tau\phi}{\rho})^{1-\eta} - f_x \right) - \alpha_x, \quad (22)$$

By assuming identical trading nations, we know that $Y^F = Y^H$, and

$$\left(P^F\right)^{1-\eta} = (P)^{1-\eta} =$$
$$\int_0^{\phi^*} N^H n_d(\phi)^{\frac{(\eta-1)}{(\sigma-1)}} (\frac{\phi}{\rho})^{1-\eta} dG(\phi|\phi < \phi^*) + \int_0^{\phi_x^*} N^F n_x(\phi)^{\frac{(\eta-1)}{(\sigma-1)}} (\frac{\tau\phi}{\rho})^{1-\eta} dG(\phi|\phi < \phi_x^*)$$

where ϕ^* and ϕ_x^* are respectively the cutoff marginal costs for domestic production and exports, which will be solved endogenously. N^H and N^F denote the number of surviving firms who actually service domestic and foreign markets. $n_d(\phi)$ and $n_x(\phi)$ denote the scope of products for domestic and foreign markets, given marginal cost ϕ. $G(\phi)$ is the distribution of marginal costs, which is assumed to be identical in both countries. The assumption of identical countries greatly simplifies our analysis and excludes the feedback effect of wage in a more general aymmetric case[14].

The first order condition to (22) gives the function for exporting varieties:

$$n_x(\phi) = \left(\frac{\eta - 1}{\sigma(\sigma - 1)} \frac{Y^H P^{\eta-1}}{f_x} (\frac{\tau\phi}{\rho})^{1-\eta} \right)^{(\sigma-1)/(\sigma-\eta)} \tag{23}$$

The least productive firm who actually exports abroad must satisfy zero export profit condition which leads to the cutoff scope condition for export:

$$n_x(\phi_x^*) = \frac{\eta - 1}{\sigma - \eta} \frac{\alpha_x}{f_x}. \tag{24}$$

where ϕ_x^* represents the cutoff exporting firm's marginal cost. This is analogous to our cutoff scope for domestic sales in equation (13). So the cutoff exporting firm's scope expands with the beachhead fixed cost of export relative to the variety-level fixed export cost (i.e., α_x/f_x). The cutoff marginal cost ϕ_x^* is endogenously determined in (23). Furthermore, from (23) for any active exporting firm with marignal cost $\phi < \phi_x^*$, its optimal number of varieties for export can be expressed as:

$$\frac{n_x(\phi)}{n_x(\phi_x^*)} = \left(\frac{\phi}{\phi_x^*} \right)^{-(\eta-1)(\sigma-1)/(\sigma-\eta)}. \tag{25}$$

[14] Another way to fix wage is to assume a numeraire homogenous good, which could be freely transported across border (Helpman, Melitz and Yeaple 2004, Baldwin and Forslid 2004). In this set-up, countries do not necessarily need to be identical.

On the other hand, comparing (12) and (23), for the same firm, its domestic scope relative to its export scope can be expressed as:

$$\frac{n_d(\phi)}{n_x(\phi)} = \left(\frac{\tau^{\eta-1} f_x}{f_d}\right)^{(\sigma-1)/(\sigma-\eta)}. \qquad (26)$$

To ensure firms do not innovate new varieties purely for export, the *regularity condition* $\tau^{\eta-1} > \frac{f_d}{f_x}$ is imposed. Higher trade costs, either the iceberg transportation costs or the beachhead fixed investment, discourage firms to expand the full range of varieties for export.

Because of the existence of fixed costs of export, exporting firms would not export all their available varieties, instead, they only export a fraction. Interestingly, once a firm starts to export, the proportion of its range of export variety relative to its total product range is fixed, independent of firm's productivity level, as shown by (26) which does not depend on ϕ. This could be due to identical costs in producing each of the firm's total varieties[15].

Firms self-select into nonproducer, domestic seller and exporter, according to their marginal cost draw upon entry. The partition by production and export status is given by (13) and (24). Clearly, the cutoff numbers of varieties for production and for export are functions of exogenous parameters. However, this does not mean the cutoff firms' productivity level is predetermined by parameters. Indeed, the cutoff productivities for firms entering the domestic market and the foreign market are endogenously determined by the zero-cutoff-profit condition and the free entry condition.

More productive firms not only produce a greater scope of varieties, but also tend to export a greater range of varieties (*the extensive margin of export*). The elasticity of the product range ratio with respect to their productivity ratio is $(\eta - 1)(\sigma - 1)/(\sigma - \eta)$. On the other hand, a more productive firm also produces and exports larger quantities in each of its product (*the intensive margin of export*), this could be seen by comparing $q_x(\phi_1)$ and $q_x(\phi_2)$ with $\phi_1 < \phi_2 \leq \phi_x^*$:

$$\frac{q_x(\phi_1)}{q_x(\phi_2)} = \left(\frac{\phi_2}{\phi_1}\right) > 1 \qquad (27)$$

[15] Eckel and Neary (2006) propose a multiproduct firm model in which a firm has a "core compentence", and is less efficient the far away from this core variety. Then a firm shall put more weight in exporting its "core competence" in which it has expertise, rather than those "periphery" varieties.

Proposition 6 *Each exporting firm exports a fixed share of its total number of varieties. More productive firms tend to export more in each of their exported varieties (the intensive margin), and they also tend to export a greater range of varieties (the extensive margin).*

Though a more productive exporting firm exports more in quantity in each of its varieties, it has the same export value in each varieties as a less productive exporting firm. The size advantage it has over the less productive counterpart, totally comes from its ability to extend product range.

More productive firms dominate less efficient firms in both domestic market and foreign market. They play more important role in international trade, because they not only export a larger amount in each exportable variety on average, but also export a greater range of varieties.

The analysis of exporting firms' total quantity ratio and total revenue ratio are analogous to (16) and (17). More productive *exporting* firms produce much greater total quantity and total revenue (including domestic and foreign sales) relative to less efficient *exporting* firm (with an elasticity of $1 + (\eta - 1)(\sigma - 1)/(\sigma - \eta)$ and $(\eta - 1)(\sigma - 1)/(\sigma - \eta)$ respectively). Comparing an exporting firm with a non-exporting domestic producer, the magnification of productivity inequality is even more pronounced.

For an exporting firm with marginal cost ϕ, its profit from domestic sales is given in (18), and its profit from export sales is,

$$\Pi_x(\phi) = \left[\left(\frac{\phi}{\phi_x^*} \right)^{-(\eta-1)(\sigma-1)/(\sigma-\eta)} - 1 \right] \alpha_x \tag{28}$$

Accordingly, *the free entry condition* in equilibrium requires each potential entrant's expected operating profit equals their sunk costs of entry:

$$E(\Pi) = \int_0^{\phi^*} \left[\left(\frac{\phi}{\phi^*} \right)^{-\frac{(\eta-1)(\sigma-1)}{(\sigma-\eta)}} - 1 \right] \alpha_d dG(\phi) + \int_0^{\phi_x^*} \left[\left(\frac{\phi}{\phi_x^*} \right)^{-\frac{(\eta-1)(\sigma-1)}{(\sigma-\eta)}} - 1 \right] \alpha_x dG(\phi) = f_e \tag{29}$$

Solving (29) gives the cutoff marginal cost for domestic production and for export[16]:

$$\left(\frac{\phi^*}{\overline{\phi}}\right)^\gamma = \kappa\frac{f_e}{\alpha_d}\left(1-\theta(\tau)\right); \quad \text{and} \quad \left(\frac{\phi_x^*}{\overline{\phi}}\right)^\gamma = \kappa\frac{f_e}{\alpha_x}\theta(\tau) \tag{30}$$

where $\theta(\tau) = \left[1+\left(\frac{\alpha_x}{\alpha_d}\right)^{\left(\frac{\gamma(\sigma-\eta)}{(\eta-1)(\sigma-1)}-1\right)}\left(\frac{f_x}{f_d}\right)^{\frac{\gamma}{\sigma-1}}\tau^\gamma\right]^{-1}$,and $\kappa = \left(\frac{\gamma(\sigma-\eta)}{(\eta-1)(\sigma-1)}-1\right)$.

$\theta(\tau)$ could be interpreted as a measure of trade convenience. Larger $\theta(\tau)$ represents better trade environment, which stems from lower relative benchhead cost of exports $\left(\frac{\alpha_x}{\alpha_d}\right)$, lower relative fixed cost of exporting an variety $\left(\frac{f_x}{f_d}\right)$, or lower ice-berge trade cost τ.

Analyze cutoff conditions and firms' scale and scope condition, we reach the following conclusion:

Proposition 7 *Trade liberalization (a reduction in τ) leads to three effects: a). selection effect: more firms start to export but fewer firms survive; b). scope effect: exporting firms expand their range of exported varieties, while rationalize on the total range of products; c).scale effect: exporting firms' on average produce more in their varieties.*

Proof. *See the Appendix.* ■

Intuitively, lower trade cost attracts new entry of firms to export abroad. Those new entrants, though relatively less productive than incumbent exporting firms, are indeed relative more productive than those who remain as local sellers. This reallocation of resources among firms tends to push up the demand for labor and therefore the real wage rises, which in turn makes unprofitable the production of those least productive local producers. In short, trade liberalization makes it easier for firms to export, while at the same time makes it harder for firms to survive. This interaction reflects interesting patterns of trade in terms of extensive margin. "Selection" of firms occurs when trade cost drops.

However, in a multiproduct framework, there is not only an *extensive margin* across firms (i.e., number of firms), but also an *extensive margin* within each exporting firm (i.e., number of varieties by each firm). How then does trade liberalization affect within-firm selection

[16] To ensure active firms do self-select into exporters and non-exporters, exporting should be relatively harder than domestic business, i.e., $(\alpha_x/\alpha_d)^{(\sigma-\eta)/(\eta-1)(\sigma-1)}\,(f_x/f_d)^{1/(\sigma-1)}\tau > 1$

over different varieties (the scope effect)? From (14) and (25), ceteris paribus, increasing ϕ^* raises the range of products sold domestically, while increasing ϕ_x^* raises the range of products sold to foreign market. However, trade liberalization forces ϕ^* and ϕ_x^* move in opposite directions. Therefore given an active exporter's productivity (inverse of marginal cost), a lower transport cost of export would lead to a shorter range of local products, but a longer range of export products. This provides a theoretical support to the Eastman-Stykolt claim that after trade liberalization, firms would rationalize their product lines. And from (26), lower trade costs lead exporting firms export a higher fraction of their products. This is the "scope effect" of trade liberalization. The adjustment in industry-level extensive margin due to firm entry and exit, as well as the adjustment in firm-level extensive margin due to variety creation and destruction, is plotted in Figure 1.

Furthermore, in this model we regain the "scale effect" which is absent in Krugman (1980): that is, the exporting firms, on average produce more per variety. It is, however, important to note that for each incumbent variety, the quantity of either domestic sales or exports does not change after trade cost reduces. Instead, the "scale effect" comes from the other two sources: first, new firms enter export market so for those firms, their quantities for each exporting variety increase; second, the incumbent exporting firm expand their export variety range so those newly exported varieties also enjoy an increase in scale. If we look at each firm by average scale per variety, the firm entry and variety entry both contribute to the expansion in scale; and finally since trade liberalization leads to fewer local producers and more exporters, the industry average firm scale also increases. However, our model also predicts no adjustment in scale for those non-exporters.

Analogous to *Proposition 5* in the closed economy case, in an open economy, we have:

Proposition 8 *In the open economy with heterogenous multiproduct firms, (1). the borderline domestic firm's scope $n_d(\phi^*)$ increases in the beachhead fixed costs for domestic market relative to the fixed cost for each variety α_d/f_d; (2). the borderline exporting firm's scope $n_x(\phi_x^*)$ increases in the beachhead fixed costs for foreign market relative to the fixed cost for each variety α_x/f_x; (3). the borderline domestic and exporting firm's cutoff marginal costs*

ϕ^* and ϕ_x^* increase in the sunk entry cost f_e; (4). firm level characteristics (i.e., firm scale and scope) are independent of market size.

We finally pin down the equilibrium number of firms who produce and who export. By the assumption of symmetric countries , the number of exporting firms from home countries equals the number of foreign firms selling in home market. By the law of large numbers, we have:

$$\frac{N^H}{N^F} = \frac{G(\phi^*)}{G(\phi_x^*)} = \left(\frac{\phi^*}{\phi_x^*}\right)^\gamma \tag{31}$$

Solving the market clearing conditions[17], we get:

$$N^H = \zeta \frac{Y}{\alpha_d} (1 - \theta(\tau)), \quad N^F = \zeta \frac{Y}{\alpha_x} \theta(\tau)$$

where we recall $\theta(\tau) = \left[1 + \left(\frac{\alpha_x}{\alpha_d}\right)^{\left(\frac{\gamma(\sigma-\eta)}{(\eta-1)(\sigma-1)}-1\right)} \left(\frac{f_x}{f_d}\right)^{\frac{\gamma}{\sigma-1}} \tau^\gamma\right]^{-1}$ which is decreasing in trade cost τ, and $\zeta = \frac{\gamma(\sigma-\eta)-(\eta-1)(\sigma-1)}{\gamma\sigma(\sigma-1)}$.

Proposition 9 *A drop in trade cost τ leads to a decrease in the equilibrium number of active firms, and an increase in the equilibrium number of exporting firms.*

Proof. *directly from equations on N^H and N^F.* ■

A drop in trade cost τ, on the one hand, makes it profitable for some less productive firms to become exporters, on the other hand, forces some least productive firm who do not export exit the market. This is best decribed in (30). This process raises the average productivity of surviving firms, but as a result allows fewer firms active in equilibrium.

The consumer's welfare, doubtlessly increases. The total number of varieties available to a consumer M, could be written as:

$$M = \underbrace{N^H \int_0^{\phi^*} n_d(\phi) dG(\phi|\phi < \phi^*)}_{\text{varieties from domestic producers}} + \underbrace{N^F \int_0^{\phi_x^*} n_x(\phi) dG(\phi|\phi < \phi_x^*)}_{\text{varieties from foreign producers}} \tag{32}$$

Substitute (14), (25) and (31) into (32), we get:

$$M = \frac{\eta - 1}{\sigma(\sigma - 1)} Y \left(\frac{1 - \theta(\tau)}{f_d} + \frac{\theta(\tau)}{f_x}\right)$$

[17] See appendix for procedure.

So a decrease in trade cost τ leads to a decrease in the total number of domestically produced varieties, and an increase in the total number of imported varieties; which one plays a dominant role depends on the magnitutde between f_d and f_x:

$$\frac{\partial M}{\partial \tau} = (\frac{1}{f_x} - \frac{1}{f_d})\frac{\partial \theta(\tau)}{\partial \tau}$$

So as pointed out precisely in Baldwin and Forslid (2004), there is a "anti-variety" effect — as long as the fixed cost of exporting a variety is greater than the fixed cost of selling a variety domestically (i.e., $f_x > f_d$)[18]. In contrast, the total number of varieties would increase when the fixed cost of selling a variety abroad is not very large relative to the fixed cost of selling it domestically, (i.e., $\tau^{\eta-1} > \frac{f_d}{f_x} > 1$), taking into account of the regularity condition $\tau^{\eta-1} > \frac{f_d}{f_x}$ which is required so that firms do not innovate varieties purely for export.

5. CONCLUDING REMARKS

The paper has constructed a simple model of multiproduct firms and discussed the impact of trade liberalization between two identical countries. Conventional monopolistic competition models, which assume single-product firms and symmetric costs across firms, do not satisfatorily address firms' response to trade liberalization in product scale and scope. Because of the dominance of multiproduct firms in international trade (Bernard, Redding and Schott 2006a) and the prevalence of heterogeneity in productivity across firms (Melitz 2003), it is of both theoretical and empirical importance to study a model which incorporates both features. In this paper, we first try to introduce international trade into Allanson and Montagna's multiproduct model with symmetric costs. In an open economy with two identical countries, reductions in trade costs lead all firms to expand their export scope but contract their total product scope. So there is rationalization within each firm after trade

[18] Melitz (2003) does note this possible "anti-variety" effect. As he points out: " when the export costs are high, these foreign firms replace a larger number of domestic firms." He then proves that although total number of varieties may decrease, the consumer welfare will be unambiguously increased: the positive gain in productivity always dominate the reduction in variety number.

liberalization.

We then introduce the Melitz-type heterogeneity into the multiproduct model. Firms differ in their marginal costs of production and the threshold entry level into both domestic and foreign markets are endogenized. With heterogenous productivity, firms self select into exporters, non-exporters and non-producers. Firms endowed with higher productivity not only sell more in each variety, but also sell a greater number of varieties. Only the most productive firms will find it profitable to export abroad, and they will only export a fixed proportion of their products. The proportion of exporting varieties depends on trade costs. Reductions in trade cost τ, will lead exporting firms to expand export range but also to contract on total number of products. This reaction by trimming product scope raises the average production scale of the exporting firms. But there is barely any change in scale for the non-exporters. The least productive firms do not export and would exit when trade costs drop. Trade liberalization leads to fewer firms surviving in equilibrium, while a higher fraction would serve the export market.

Our theoretical model can be well connected to the Canadian experience after the Canada-U.S. Free Trade Agreement in 1989. In particular, the Eastman-Stykolt hypothesis (Eastman and Stykolt 1967) suggests that in a small open economy after trade liberalization, firms would be larger in scale and manage fewer number of varieties. Using a large plant-level panel data, Baldwin and Gu (2005) find that exporters reduce product diversification, and increase production-run length and plant size, but they conclude that those changes are responses to forces other than tariff cuts. Our model provides an alternative theoretical model which is based on the conventional Dixit-Stiglitz monoplistic competition model. Our model helps to address those interesting points found in the empirics. Though the model does find adjustment in scale and scope within each exporting firm, it remains uncovered for the inner mechanism of within firm selections since we assume symmetry among varieties within a firm. We also do not tackle the possible interaction of country size with firm size, which is left for future research.

APPENDIX

Proof on the number of active firms N^H and exporting firms N^F (footnote 14).—

Proof. We start from the cutoff conditions for the domestic producing firms (12), and (13), which can be modified as:

$$n_d(\phi^*)^{(\sigma-\eta)/(\sigma-1)} = \left(\frac{\eta-1}{\sigma-\eta}\frac{\alpha_d}{f_d}\right)^{(\sigma-\eta)/(\sigma-1)} = \frac{\eta-1}{\sigma(\sigma-1)}\frac{Y^H}{f_d P^{1-\eta}}(\frac{\phi^*}{\rho})^{1-\eta}, \quad (33)$$

where the price index P could be solved in the following expression:

$$\frac{(P)^{1-\eta}}{(\frac{\phi^*}{\rho})^{1-\eta}} = \int_0^{\phi^*} N^H n_d(\phi)^{\frac{(\eta-1)}{(\sigma-1)}}(\frac{\phi}{\phi^*})^{1-\eta}dG_1(\phi) + \int_0^{\phi_x^*} N^F n_x(\phi)^{\frac{(\eta-1)}{(\sigma-1)}}(\frac{\tau\phi}{\phi^*})^{1-\eta}dG_2(\phi),$$

where $G_1(\phi) = G(\phi|\phi < \phi^*) = (\frac{\phi}{\phi^*})^\gamma$ is the conditional CDF for firms staying in the domestic market, while $G_2(\phi) = G(\phi|\phi < \phi_x^*) = (\frac{\phi}{\phi_x^*})^\gamma$ is the conditional CDF for firms exporting abroad.

Use (13), (14), (24), (25), and (31), and let $k_1 = \left(\frac{\eta-1}{\sigma-\eta}\frac{\alpha_d}{f_d}\right)^{(\eta-1)/(\sigma-1)}$, $k_2 = \left(\frac{\eta-1}{\sigma-\eta}\frac{\alpha_x}{f_x}\right)^{(\eta-1)/(\sigma-1)}$, it can be rewritten as:

$$\begin{aligned}
\frac{(P)^{1-\eta}}{(\frac{\phi^*}{\rho})^{1-\eta}} &= N^H \int_0^{\phi^*} k_1(\frac{\phi}{\phi^*})^{-\frac{(\sigma-1)(\eta-1)}{(\sigma-\eta)}}dG_1(\phi) + N^F \int_0^{\phi_x^*} k_2(\frac{\phi}{\phi_x^*})^{-\frac{(\sigma-1)(\eta-1)}{(\sigma-\eta)}}(\frac{\tau\phi_x^*}{\phi^*})^{1-\eta}dG_2(\phi) \\
&= \frac{\gamma}{\gamma-\frac{(\eta-1)(\sigma-1)}{(\sigma-\eta)}}N^H k_1\left[1+\left(\frac{\alpha_x}{f_x}\frac{f_d}{\alpha_d}\right)^{\frac{(\eta-1)}{(\sigma-1)}}(\frac{\phi_x^*}{\phi^*})^{\gamma+1-\eta}\tau^{1-\eta}\right] \\
&= \frac{\gamma}{\gamma-\frac{(\eta-1)(\sigma-1)}{(\sigma-\eta)}}N^H k_1\left[1+\left(\frac{\alpha_x}{f_x}\frac{f_d}{\alpha_d}\right)^{\frac{(\eta-1)}{(\sigma-1)}}(\frac{\alpha_d}{\alpha_x}\frac{\theta(\tau)}{1-\theta(\tau)})^{\frac{\gamma+1-\eta}{\gamma}}\tau^{1-\eta}\right],
\end{aligned}$$

plugging this back to 33, and using $\theta(\tau) = \left[1 + \left(\frac{\alpha_x}{\alpha_d}\right)^{\left(\frac{\gamma(\sigma-\eta)}{(\eta-1)(\sigma-1)}-1\right)} \left(\frac{f_x}{f_d}\right)^{\frac{\gamma}{\sigma-1}} \tau^{\gamma}\right]^{-1}$, we will get the solution for N^H and N^F

$$N^H = \frac{\gamma(\sigma-\eta)-(\eta-1)(\sigma-1)}{\gamma\sigma(\sigma-1)} \frac{Y}{\alpha_d} (1-\theta(\tau))$$

$$N^F = \frac{\gamma(\sigma-\eta)-(\eta-1)(\sigma-1)}{\gamma\sigma(\sigma-1)} \frac{Y}{\alpha_x} \theta(\tau)$$

■

Proof of Proposition 7.—

Proof. Since $\frac{\partial\theta(\tau)}{\partial\tau} < 0$ and $\frac{\partial\theta(\tau)}{\partial(f_x/f_d)} < 0$, we have $\frac{\partial\phi^*(\tau)}{\partial\tau} > 0$, and $\frac{\partial\phi_x^*(\tau)}{\partial\tau} < 0$. Furthermore $\frac{\partial\phi^*}{\partial(f_x/f_d)} > 0$, $\frac{\partial\phi_x^*}{\partial(f_x/f_d)} < 0$. Therefore, there are firm level selections occur in response to a decrease in transport cost τ or a decrease in benchhead fixed cost of export f_x/f_d. ■

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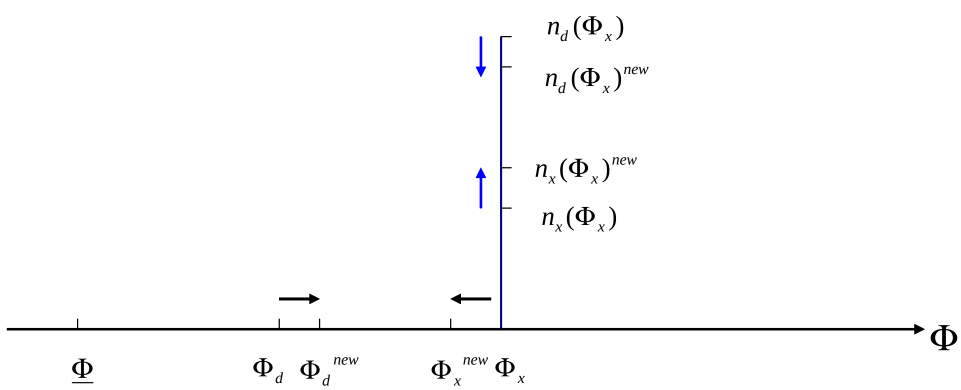

Trade liberalization (a reduction in τ) leads to :

- a). selection effect: more firms start to export but fewer firms survive;
- b). scope effect: exporting firms expand their range of exported varieties, while rationalize on the total range of products;