

WHO SHRUNK CHINA? PUZZLES IN THE MEASUREMENT OF REAL GDP*

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The latest World Bank estimates of real GDP per capita for China are significantly lower than previous ones. We review possible sources of this puzzle including substitution bias in consumption, reliance on urban prices, which we estimate are higher than rural ones, and the use of an expenditure-weighted rather than an output-weighted measure of GDP. Taking all these together, we estimate that Chinese real per capita GDP was 30% higher in 2005 than the World Bank estimates. Our empirical procedures have implications more broadly for international comparisons of living standards and real GDP.

The International Comparison Programme (ICP) is a collaborative effort of the World Bank and other international agencies to estimate the ‘real GDP’ of countries, that is, the value of their GDP when adjusted for price-level differences across countries and (by convention) priced in US dollars. Because market exchange rates cannot be relied upon to provide the right conversion from national currencies to dollars, the ICP computes ‘purchasing power parity’ (PPP) exchange rates, which compare local prices of a basket of goods with the US prices of the same basket in a benchmark year. For China, the 2005 benchmark estimates from the World Bank show that real GDP per capita for China was 40% smaller in 2005 than real GDP for the *same* year based on extrapolations from earlier rounds of the ICP. As Deaton and Heston (2010, p. 3) report:

the 2007 version of the World Development Indicators (WDI), World Bank (2007), lists 2005 per capita GDP for China as \$6,757 and for India as \$3,452, both in current international dollars. The 2008 version, World Bank (2008*a*), which includes the new [2005] ICP data, gives, for the same year, and the same concept \$4,088 for China and \$2,222 for India. For comparison, GDP per capita at market exchange rates is \$1,721 for China and \$797 for India.

Maddison (2007) argues that such a downward revision for China is implausible because extrapolating backwards it would imply per capita income below subsistence

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levels in early years.¹ This observation raises the questions of why the downward revision occurred and whether alternative calculations would give noticeably different results. The first of these questions has been addressed by Heston (2007) and Deaton and Heston (2010).² We will provide some theoretical structure to help understand their critique, and then address the second question: whether alternative, theoretically consistent calculations of real GDP make a difference for China or other countries. While our primary focus is on the 'China puzzle', our article illustrates some of the pitfalls and potential of alternative methods of making international comparisons of real consumption and GDP.

Among the reasons provided by Deaton and Heston as to why the relative position of China was lowered in the most recent ICP round are the following:

- (i) The price data provided by China for the most recent ICP was for urban areas, and may have overstated the actual prices faced by rural consumers.

This first issue is a well-recognised feature of the price data provided for the 2005 ICP, which was the first time that China participated fully in the round.³ Deaton and Heston note that there is a theoretical issue of aggregating over large countries with diverse prices between regions but we do not address that issue here. Rather, we treat the urban bias in the prices reported by China to the 2005 ICP as an empirical issue to be investigated. The Penn World Table version 7.0 (PWT 7.0) introduced a 20% uniform downward adjustment to the ICP prices of consumption goods for China, reflecting the urban bias.⁴ In Section 1, we investigate the extent to which that adjustment is justified by comparison with prices for other developing Asian countries.

- (ii) Different index number methods imply different relative sizes of countries.

There are very substantial differences in the index number methods used by the World Bank and by the Penn World Table (PWT). We focus here on just one aspect of those differences, the contrast between the fixed-weight indexes used in PWT and the flexible-weight index used by the ICP/World Bank. Neary (2004) has proposed an alternative that requires estimating the expenditure function across countries to obtain indexes of real consumption. This approach was further developed by Feenstra *et al.* (2009b) and empirically implemented for data covering 124 countries. In Section 1, we use all these methods, along with a recommendation by Barnett *et al.* (2009), to calculate the size of real consumption in China. In addition to showing the differences between the various index number approaches, we use a number of different sets of prices for China: the prices

¹ Using the Chinese national accounts growth rates would imply a real per capita GDP in 1970 of \$400 at constant 2005 prices. The corresponding figure at the current 1970 prices is \$93.

² See also the comments by Diewert (2010a).

³ See Asian Development Bank (2007) for more details regarding price data from China used in the 2005 ICP Asia-Pacific region. It is clear that urban bias would have impacted price data for consumption items.

⁴ As explained in Section 3, PWT 7.0 produced two estimates of real GDP for China: one using the 'official' ICP prices and the second using the 20% downward adjustment in prices. The same procedure is used in PWT 7.1, which differs from 7.0 based on revised national accounts data and revised prices for investment goods.

they reported to the 2005 ICP and our own estimates of alternative prices from several empirical models using prices from other Asian countries. We find that the impact of adjusting China's prices is quite large: real consumption in China is 8%–28% higher using our adjusted prices than using the ICP prices. The higher estimates are obtained using flexible-weight indexes of consumption and our preferred estimates of alternative prices. Our preferred results are therefore very similar to what is found from the 20% uniform downward adjustment to consumption prices used by PWT 7.0, which corresponds to a 25% increase in real consumption.

Closely related to the index number issue but conceptually distinct is:

- (iii) There are several different concepts of real GDP that can be measured, each of which can imply different relative sizes of countries.

Shifting attention from real consumption to real GDP, in Section 2 we incorporate investment, government spending and the trade balance. We draw on estimates of real GDP from Feenstra *et al.* (2013), which provides the basis for the 'next generation' of PWT version 8.0.⁵ Preliminary estimates using PWT 8.0 give real GDP per capita in China for 2005 at \$4,749 using ICP prices, which is about 15% larger than the estimate of \$4,088 from the World Bank. If the prices of consumption goods are adjusted downwards by 20%, then real GDP per capita rises to \$5,349, or another 15% higher than the initial estimates by the World Bank. We conclude that the World Bank's estimate is too low by as much as 30%.

Turning to alternative concepts of GDP, Feenstra *et al.* (2009a) have recently contrasted real GDP measured on the expenditure side of the economy, as done by the World Bank and PWT, with real GDP measured on the output side. These two concepts differ by countries' terms of trade, that is, by the price of their exports relative to imports. PWT 8.0 will provide calculations of real GDP measured on the output side taking into account the terms of trade. We find that this new calculation makes only a modest difference to China's real GDP in 2005: measured at ICP prices, it falls to \$4,694, and measured at reduced consumption prices, it falls to \$5,279, or a fall of less than 1.5% in both cases.

In Section 3, we explain that these modest differences from China arise because its terms of trade are below the world average but, in addition, it is running a trade surplus. The former tends to lower real GDP on the expenditure side but the latter tends to raise it above real GDP on the output side, so the net effect is the modest difference that we measure.

However, we argue that if real GDP were corrected for substitution bias using a revenue function estimated across countries, analogous to the expenditure function approach to real consumption of Neary (2004), then China's real GDP could be higher than our measure. We conclude that while real GDP in China relative to the US is quite plausibly 30% higher than estimated by the World Bank (2008a), higher estimates could be obtained by future implementation of flexible-weight methods on the

⁵ PWT 8.0 and PWT 7.0 both use the ICP benchmark year of 2005 but are given different numbers because 8.0 will be produced by the University of California, Davis and the University of Groningen, whereas 7.0 was produced by the University of Pennsylvania (and 7.1 is the last version to be released by Penn).

production side. Additional empirical results are in Appendix A and B, and the proofs of propositions are in Appendix C.

1. Real Consumption

In Table 1 and the Appendix Table A1, we show various calculations of real consumption using data for 124 countries from the 2005 ICP.⁶ There are 12 categories of consumption goods, listed in Appendix Table A2, which we aggregate to compute real consumption.⁷ We report results for selected countries in Table 1, ranked by their levels of nominal consumption in US\$ per capita in column (1), with results for all 124 countries shown in Appendix Table A1. The summary statistics at the bottom of Table 1 refer to the whole sample of countries. All indexes are given relative to the US.

1.1. Fixed and Flexible-weight Indexes

The first multilateral index in Table 1, column (2) is the GK (Geary, 1958; Khamis, 1970, 1972) system, as used by the PWT, and applied here to the $i = 1, \dots, M$ final consumption goods, with prices p_{ij} and quantities q_{ij} across countries $j = 1, \dots, C$.⁸ The reference prices (denoted by e for expenditure) π_i^e and the purchasing power parities PPP_j^e are defined as the solution to the simultaneous system:

$$\pi_i^e = \frac{\sum_{j=1}^C \left(\frac{p_{ij}}{PPP_j^e} \right) q_{ij}}{\sum_{j=1}^C q_{ij}}, \quad i = 1, \dots, M, \quad (1)$$

$$PPP_j^e = \frac{\sum_{i=1}^M p_{ij} q_{ij}}{\sum_{i=1}^M \pi_i^e q_{ij}} \quad j = 1, \dots, C, \quad (2)$$

subject to a normalisation. The PPPs in (2) are used to adjust expenditure in national currency to obtain that in reference prices, or real consumption:

⁶ The total number of countries in the 2005 ICP is 146 but we omitted 22 countries with missing expenditures or prices for some consumption goods, or for other data reasons. Details are available on request.

⁷ As the current Section focuses on private consumption only, we make use of 12 categories for individual consumption and disregard other components: individual consumption expenditure by government; collective consumption expenditure by government; and gross fixed capital formation. Necessary data are downloaded from World Bank (2008b).

⁸ It should be noted that goods here are composite commodities, corresponding to the 12 aggregates listed in Table A2. As a result, the prices, p_{ij} refer to the price index for commodity group i in country j with US as the base country and the quantities, q_{ij} refer to real expenditures for different commodity groups obtained by deflating nominal expenditures by the price indexes, p_{ij} .

Table 1
Real Consumption for Selected Countries, 2005 (US = 100)

Countries	Nominal consp. per capita (1)	Consumption indexes				Using AIDS expenditure function and reference prices from:		
		Geary- Khamis (2)	Fisher (3)	GEKS (4)	CCD (5)	Geary- Khamis (6)	Neary, GAIA (7)	Diewert (8)
US	100	100	100	100	100	100	100	100
% Diff from GEKS		0.00	0.00	0.00	0.00	0.00	0.00	0.00
UK	89.5	77.5	76.6	79.3	80.7	77.9	76.7	77.9
% Diff from GEKS		-2.27	-3.40	0.00	1.77	-1.77	-3.28	-1.77
Ireland	84.3	64.0	65.4	66.2	64.8	66.4	64.8	66.5
% Diff from GEKS		-3.32	-1.21	0.00	-2.11	0.30	-2.11	0.45
Netherlands	75.6	70.6	70.2	72	73.6	69.8	68.2	69.9
% Diff from GEKS		-1.94	-2.50	0.00	2.22	-3.06	-5.28	-2.92
Canada	73.1	72.9	73.1	73.5	73.3	74	72.7	74.1
% Diff from GEKS		-0.82	-0.54	0.00	-0.27	0.68	-1.09	0.82
Korea, Rep.	29.7	38.0	38.9	37.5	37.8	38.2	36	38.3
% Diff from GEKS		1.33	3.73	0.00	0.80	1.87	-4.00	2.13
Macedonia	7.6	19.8	18.8	19.2	19.5	19.4	17.6	19.6
% Diff from GEKS		3.13	-2.08	0.00	1.56	1.04	-8.33	2.08
China	2.3	6.0	6.0	5.6	5.7	5.4	4.5	5.4
% Diff from GEKS		7.14	7.14	0.00	1.79	-3.57	-19.64	-3.57
India	1.4	4.9	4.7	4.6	4.7	4.7	3.9	4.7
% Diff from GEKS		6.52	2.17	0.00	2.17	2.17	-15.22	2.17
Mean of % diff from GEKS		2.330	0.211	0.000	0.833	-1.085	-11.727	-0.405
SE of the mean difference		(0.54)	(0.449)	(0.00)	(0.191)	(0.409)	(0.830)	(0.402)
Regression of index on GEKS		0.981	0.990	1.000	1.005	1.021	1.081	1.017
SE of slope coefficient		(0.004)	(0.005)	(0.000)	(0.002)	(0.004)	(0.005)	(0.004)

Notes. Each row reports indexes for real consumption expenditure for selected countries relative to the US, followed by a row showing the percentage difference of each index relative to the GEKS for the same country. Column (1) gives the nominal consumption expenditure (obtained using exchange rates for converting national currency data into US dollars) per capita in 2005, relative to the US. Column (2) gives the Geary-Khamis calculation of real consumption, while column (3) gives the bilateral Fisher index relative to the US. Columns (4)–(5) present flexible weight measures of real consumption expenditure per capita, using, respectively, the GEKS and CCD methods, as defined in the text in equations (4)–(5). Columns (6)–(8) show measures of real consumption based on the estimated AIDS expenditure function, using different reference prices: column (6) uses the Geary-Khamis reference prices; column (7) uses the GAIA reference prices; and column (8) uses every country's prices as reference and then takes the geometric mean of the results, as in Barnett *et al.* (2009). The last four rows are for the sample of 124 countries, where the last two rows report the slope coefficient (and standard error) from a regression of the natural log of each index on the log of the GEKS index. AIDS is almost ideal demand system; GAIA is Geary-Allen international accounts.

Source. Authors' calculations using 2005 ICP benchmark program data.

$$\sum_{i=1}^M \pi_i^e q_{ij} = \frac{\sum_{i=1}^M p_{ij} q_{ij}}{PPP_j^e}, \quad j = 1, \dots, C. \quad (3)$$

This index number system has the unique advantage that it aggregates consistently across countries and commodities. On the other hand, real consumption for different countries computed as in (3) uses fixed quantities q_{ij} , implying that consumers do not respond to price changes, so we refer to it as a ‘fixed-weight’ index.

Column (2) of Table 1 shows that GK real consumption in China is only 6.0% of that in the US in 2005. This estimate is more than twice as large as what we get from comparing nominal consumption in US dollars using official exchange rates in column (1), but is much smaller than the estimate of 9.8% for total real GDP per capita (including C, I, G and X–M) relative to the US from the 2005 ICP (World Bank, 2008a),⁹ let alone the estimate of 18.5% for real GDP per capita that Maddison (2007, Table 5) claims is needed to avoid having Chinese living standards below subsistence in past decades. So the conclusion is that per capita real consumption is low for China, relative to other countries or relative to its GDP. We now investigate whether this finding depends on the index number method that is used to compute real consumption.

A drawback with any fixed-weight index number such as GK is that it fails to allow for substitution effects in consumption. As a result, it risks introducing a bias in international comparisons, the ‘Gerschenkron (1951) effect’, whereby any country j ’s real consumption level is overestimated when it is evaluated using the prices of any other country, as we explain below.¹⁰ In columns (3)–(5), we report various ‘flexible-weight’ indexes, so named because they use index number formulas that are known to be exact for particular specifications of preferences. As a result, they would fully offset the substitution bias of a fixed-weight index such as GK if consumption was generated by those preferences, and are often assumed to be superior to fixed-weight indexes for any underlying preferences.¹¹ The first method we report is the Fisher quantity index for each country j relative to a base country k :

$$Q_{jk}^F \equiv \left(\frac{\sum_{i=1}^M p_{ij} q_{ij}}{\sum_{i=1}^M p_{ij} q_{ik}} \right)^{0.5} \left(\frac{\sum_{i=1}^M p_{ik} q_{ij}}{\sum_{i=1}^M p_{ik} q_{ik}} \right)^{0.5}, \quad j, k = 1, 2, \dots, C.$$

The values of this index are given in column (3) of Table 1. This index is of interest in itself: Diewert (1976) showed that, like the Törnqvist index to be considered below, it is a ‘superlative’ index, in the sense that it is consistent with

⁹ As explained above, World Bank (2008a) which uses the 2005 ICP data gives real GDP per capita in China of \$4,088 in 2005, as compared with \$41,674 in the US. The ratio of these is 9.8%.

¹⁰ Samuelson (1974) summarises this principle as ‘it always looks better to ride the other fellow’s horse’, or in the words of Robert Summers, ‘the grass is greener on the other side’, meaning that real GDP tends to be higher when prices different from a country’s own are used. This pattern follows because the PPP in (2) is a Paasche price index (as noted by Deaton and Heston, 2010, p. 9), which values the quantities consumed at national prices, in the numerator. Paasche price indexes typically understate the ‘true’ difference in prices being compared (i.e. understate relative to an index that allows for substitution in consumption) and, therefore, real consumption in (3) is overstated.

¹¹ Balk (2008, 2009), Diewert (1976, 1999) and Neary (2004).

preferences that provide a local second-order approximation to arbitrary preferences. However, as with any bilateral index, using the US as base is not an innocuous normalisation: a different reference country would imply different values for the index and, probably, a different ranking of countries. To avoid this potential intransitivity, the so-called GEKS index,¹² which is used by the World Bank, takes the geometric mean of all possible bilateral comparisons to yield a transitive multilateral index:

$$Q_{jk}^{\text{GEKS}} = \prod_{l=1}^C \left(Q_{jl}^F Q_{lk}^F \right)^{1/C}, \quad j, k = 1, 2, \dots, C. \quad (4)$$

The GEKS estimates of real consumption are shown in column (4), from which we see that China is 5.6% of the US, which is even lower than in the GK system. More generally, the GK estimates of real consumption understate the GEKS estimates for rich countries and overstate the GEKS estimates for poor countries, which illustrates the Gerschenkron Effect.¹³ In particular, the GK estimates are less than the GEKS estimates for most countries with nominal GDP per capita above South Korea (ranked 31st of 124 countries), and greater than the GEKS estimates for most countries with nominal GDP per capita below Macedonia (ranked 61st of 124), with a mixed pattern in between. These two countries are included in Table 1, and we will argue below that the GK reference prices can be thought of as lying in between the prices of South Korea and the US.

An alternative flexible-weight system is the CCD (Caves *et al.*, 1982*a,b*) index, which is a multilateral generalisation of the Törnqvist index rather than the Fisher index, and as such is theoretically superior.¹⁴ The Törnqvist index of real consumption in country j relative to consumption in country k index is given by:

$$Q_{jk}^T \equiv \prod_{i=1}^M \left(\frac{q_{ij}}{q_{ik}} \right)^{\frac{s_{ij} + s_{ik}}{2}} \quad \text{where } s_{ij} = \frac{p_{ij} q_{ij}}{\sum_{i=1}^M p_{ij} q_{ij}} \quad j, k = 1, 2, \dots, C.$$

It is easy to see that these quantity indexes are not transitive unless the expenditure shares s_{ij} are the same in all the countries. The CCD index is a transitive index generated from the matrix of all bilateral Törnqvist indexes and is defined as:

¹² Sometimes called simply the EKS system, this was independently developed by Gini, Eltetö and Köves and Szulc. Rather than providing detailed historical references to the multilateral comparison methods we employ, we refer the reader to Balk (2008), which provides a modern treatment of them all. Balk (2009) suggests renaming the GEKS index the 'GEKS-Fisher' and the CCD index (which we discuss below) the 'GEKS-Törnqvist'.

¹³ As already noted, the Gerschenkron effect implies in bilateral comparisons that all countries' real consumption is overestimated when calculated using the weights of any reference country. In the GK system, the reference country is the hypothetical country whose prices equal the world prices in (1). A further complication is that, as already noted, we normalise real consumption to equal 100 for the US, so eliminating the overstatement for the US by construction. With the magnitude of the Gerschenkron effect increasing in the difference between a country's true real consumption and that of the reference country, it can be shown that this normalisation implies the pattern found in the data.

¹⁴ Diewert (1976) shows that the Törnqvist index is exact if preferences can be represented by a non-homothetic translog distance function, whereas the corresponding result for the Fisher index is that it is exact if the direct utility function is homogeneous quadratic.

$$Q_{jk}^{\text{CCD}} = \prod_{l=1}^C (Q_{jl}^T Q_{kl}^T)^{1/C} \quad j, k = 1, 2, \dots, C. \quad (5)$$

The CCD indexes are reported in column (5) of Table 1, and are quite close to the GEKS indexes. Notably, neither the Fisher nor the CCD indexes have a consistent pattern as compared with the GEKS index, as we found when comparing GK and GEKS.

At the bottom of Table 1, we report the mean of the percentage difference of each index as compared with GEKS, for the total sample of 124 countries. This mean difference is greatest for the GK system at more than 2%, smaller for the CCD index and insignificantly different from zero for the Fisher index. We also report the regression coefficient of the natural log of each index on the log of the GEKS index. We see that the GK measure of real consumption has a slope coefficient that is significantly less than unity, illustrating again the pattern whereby real consumption is overstated for poor countries and understated for rich countries in the GK system. The slope coefficients for the Fisher and CCD indexes are quite close to unity.

1.2. *Urban Price Bias*

In general, the three indexes of real consumption discussed so far – the GK, GEKS and CCD – give quite similar results especially for China. We conclude that the choice of index number method cannot account for the very low level of real consumption in China that is obtained when we use the prices collected for the 2005 ICP. But, the validity of these prices themselves is open to question. It is clear from Asian Development Bank (2007) that the price surveys in China were restricted to 11 capital cities and the rural areas surrounding these 11 cities. Following the recommendation of an Expert Group, the ADB constructed national average prices using an extrapolation method described in Asian Development Bank (2007). However, the extrapolation method made no explicit allowance for spatial price differences across different regions and between rural and urban regions of China. As a result, the general consensus is that the national average prices tend to overstate the actual prices. In particular, PWT 7.0 and Deaton and Heston (2010) assume that Chinese urban prices are uniformly 20% higher than average urban–rural prices.

Here, we attempt to take a more systematic approach to this problem. We use data for 22 developing Asian countries other than China to estimate the relationship between the 2005 ICP price level of commodity group i in country j and real per capita income in country j . We consider four models that differ in their explanatory variables:

- (i) Model 1: uses the real per capita expenditure index specific to the commodity group as the explanatory variable.
- (ii) Model 2: uses the real GDP per capita index in PPP terms as the explanatory variable.
- (iii) Model 3: uses the nominal GDP per capita index, which converts GDP in national currency units into a common currency using nominal exchange

rates, as the explanatory variable. This is because the explanatory variable chosen in model 2 can suffer from possible endogeneity as the real per capita GDP depends on the PPP for the commodity group.

- (iv) Model 4: uses the real per capita index for private consumption in PPP terms instead of the index for the whole of GDP as the explanatory variable. The main reason for considering the index for private consumption is that several countries in the Asia-Pacific region have high real GDP, but low consumption levels. In such cases, the link between price levels and the consumption index may be stronger.

Of these four model specifications, we believe that models 1 and 4 are the most appropriate. Model 1 has the benefit of using price levels specific to each commodity group, while the nature of the countries in the Asia-Pacific region makes model 4 especially appropriate. Within the Asia-Pacific region, there are a number of countries which are rich as measured by per capita GDP but have only moderate levels of consumption, comparable to those in a middle-income country. Model 4 accounts for this by considering the relationship between price levels and the real per capita index for consumption. In Appendix B, we report regression results for all four models and several variations on them. Of the 12 categories of consumption goods, we adjust five prices downwards (for food and non-alcoholic beverages; clothing and footwear; education; restaurants; and other goods and services), four prices upwards in some specifications (for gross rent, fuel, power; medical and health services; transport; and recreation) and leave three category prices unchanged due to lack of data.

Table 2
Real Consumption for China with Adjusted Prices, 2005 (US = 100)

Method of adjusting Chinese prices	Consumption indexes				Using AIDS expenditure function and reference prices from:		
	Geary–Khamis (2)	Fisher (3)	GEKS (4)	CCD (5)	Geary–Khamis (6)	Neary, GAIA (7)	Diewert (8)
20% reduction in prices	7.5	7.5	7.1	7.1	6.8	5.8	6.8
% Diff from China in Table 1	25.0	25.0	26.8	24.6	25.9	28.9	25.9
Regression model 1	7.0	6.8	6.6	6.6	6.7	5.6	6.9
% Diff from China in Table 1	16.7	13.3	17.9	15.8	24.1	24.4	27.8
Regression model 2	6.5	6.5	6.1	6.2	5.9	4.9	5.9
% Diff from China in Table 1	8.3	8.3	8.9	8.8	9.3	8.9	9.3
Regression model 3	6.4	6.4	6.0	6.1	5.8	4.8	5.9
% Diff from China in Table 1	6.7	6.7	7.1	7.0	7.4	6.7	9.3
Regression model 4	7.4	7.3	6.8	6.9	6.6	5.5	6.6
% Diff from China in Table 1	23.3	21.7	21.4	21.1	22.2	22.2	22.2

Notes. See notes to Table 1. Each row here gives the results from using adjusted consumption prices for China, and the corresponding percentage difference relative to the China estimate in Table 1. Besides adjusting all Chinese prices downward by 20%, we take four log-linear regression models to adjust Chinese prices, as described in Appendix B. Model 1 uses real per capita expenditure index specific to the commodity group as the regressor variable. Model 2 uses real GDP per capita index in PPP terms. Model 3 uses nominal GDP per capita index in a common currency. Model 4 uses real per capita index for private consumption in PPP terms instead of the index for the whole of GDP.

Source. Authors' calculations using 2005 ICP benchmark program data and Asian Development Bank (2007).

Real income comparisons based on the adjusted prices for China are presented in Table 2. In the GK calculation in column (2), under model 1, we find that real consumption in China relative to the US rises from 6% to 7%, which is a rise of 16.7%. Smaller increases are seen for models 2 and 3 but a larger increase to 7.4% is found for model 4, an increase of 23.3% compared with Table 1. Similar percentage increases are seen for the Fisher index, GEKS and CCD methods. Interestingly, the increases under model 4 are quite close to that obtained from the 20% uniform reduction in Chinese consumption prices, which corresponds to a 25% increase in real consumption. We find a somewhat lower increase in real consumption under model 1, but we will find larger increases for that model as we next consider results based on the expenditure function.

1.3. Expenditure Function Approach

While the flexible-weight indexes of subsection 1.1 have secure theoretical foundations for bilateral comparisons, these results do not extend to multilateral comparisons in the realistic case when tastes are not homothetic. To address this issue, Neary (2004), following Allen (1949), has proposed that real consumption should be measured instead using an expenditure function. This avoids the substitution bias implied by fixed-weight indexes such as GK and also allows for departures from homotheticity. The measure of real consumption at reference prices π is

$$e(\pi, u_j) = \sum_{i=1}^M \pi_i q_{ij}^*, \quad j = 1, \dots, C, \quad (6)$$

where we use an asterisk to denote optimally chosen quantities, as contrasted with the fixed quantities in (1). Real consumption in country j relative to country k is then given by,

$$\frac{e(\pi, u_j)}{e(\pi, u_k)}.$$

Neary argues that this formulation will give better estimates than a fixed-weight index number method because quantities respond to the reference prices.

Before applying the expenditure function, it is desirable to check that the data are consistent with the hypothesis of utility maximisation. It turns out that the data are extremely 'close' to satisfying consistency with the maximisation of an arbitrary utility function but very far from consistent with homothetic preferences.¹⁵ This is in line with

¹⁵ We measure consistency with maximisation of an arbitrary or a homothetic utility function by testing whether the data come 'close' to satisfying the General and the Homothetic Axioms of Revealed Preference (GARP and HARP) respectively. We measure 'closeness' using the Afriat Efficiency Index due to Afriat (1967) and Varian (1990): one minus the index is the cost, measured as a proportion of the consumer's budget, of inefficient consumption choices relative to utility maximisation. For the full sample of 124 countries, the Afriat Efficiency index for GARP is 0.9734 (implying a very small cost of inefficiency relative to utility maximisation, approximately 2.5% of the budget), whereas that for HARP is 0.6620 (implying that the cost of inefficiency relative to the maximisation of a homothetic utility function is approximately 33.4% of the budget). When three countries (Greece, Finland, Kenya) are dropped, the data satisfy GARP but HARP is still substantially violated.

the findings of Crawford and Neary (2008) for the 1980 ICP data and we conclude that the expenditure function approach can legitimately be applied to our data set.

The expenditure function approach could be implemented in many alternative ways, using alternative specifications of the expenditure function and alternative reference prices. Here, we shall use the expenditure function corresponding to the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), which is given by:¹⁶

$$\ln e(p, u) = \alpha_0 + \alpha' \ln p + \frac{1}{2} \sum_{i=1}^M \sum_{j=1}^M \gamma_{ij} \ln p_i \ln p_j + b(p) \ln u, \quad (7)$$

where $b(p) = \eta \prod_i p_i^{\beta_i}$. We impose ‘money metric scaling’, which leads to the following restrictions on the parameters: $\alpha_0 = 0$, $\eta = 1$. To ensure that the expenditure function is homogeneous of degree one in prices, we require that $\sum_i \alpha_i = 1$ and $\sum_i \beta_i = \sum_i \gamma_{ij} = 0$ for all j , and to ensure that expenditure is increasing in utility, we require that $b(p) > 0$. We report the estimated parameter values using data on 124 countries and 12 commodity groups in Appendix Table A2.¹⁷

We assume that the parameters of the expenditure function are common across countries.¹⁸ Then it follows immediately by computing $e(\pi, u_j)$ and $e(\pi, u_k)$ from (7) that the ratio of real consumption in country j relative to k is:

$$\frac{e(\pi, u_j)}{e(\pi, u_k)} = \left(\frac{u_j}{u_k} \right)^{b(\pi)} \quad j = 1, \dots, C. \quad (8)$$

For any reference prices π , we refer to (8) as a measure of real consumption based on the expenditure function, and it depends on the reference price vector π .

Consider next the choice of reference-price vector. Neary (2004) proposes that it should be computed as the solution to:

$$\pi_i^{\text{GAIA}} = \frac{\sum_{j=1}^C \left(\frac{p_{ij}}{PPP_j^*} \right) q_{ij}}{\sum_{j=1}^C q_{ij}^*} \quad i = 1, \dots, M, \quad (9)$$

$$PPP_j^* = \frac{\sum_{i=1}^M p_{ij} q_{ij}}{\sum_{i=1}^M \pi_i^{\text{GAIA}} q_{ij}^*} = \frac{e(p_j, u_j)}{e(\pi^{\text{GAIA}}, u_j)}, \quad j = 1, \dots, C, \quad (10)$$

which extends the GK system in (1)–(2) by using optimal quantities q_{ij}^* in the denominators of (9) and (10). Notice that PPP_j^* is the ratio of the expenditure

¹⁶ Estimates are also available on request for the QUAIDS expenditure function of Banks *et al.* (1997), which extends the AIDS model by adding a quadratic term in income. As in Neary (2004), this made relatively little difference in practice and also lacks the convenient theoretical properties that we exploit in (8) and subsequently.

¹⁷ These are estimated with the GAUSS program provided in Neary (2004) for estimating the AIDS and QUAIDS expenditure functions using the semiparametric approach of Diewert and Wales (1988).

¹⁸ This does not imply that preferences are actually the same in all countries, rather that we evaluate real consumption using a reference consumer whose consumption pattern comes closest to mimicking world demand patterns. See Neary (2004) for further discussion.

Table 3
Actual and Reference Prices

Category	Sample mean	US	Geary–Khamis	Neary, GAIA	South Korea
Food and non-alcoholic beverages	1	1.100	0.922	0.800	1.627
Alcoholic beverages and tobacco	1	1.395	0.872	0.587	1.162
Clothing & footwear	1	1.136	0.852	0.670	1.257
Gross rent, water, fuel and power	1	1.889	1.034	0.820	1.446
Household furnishings	1	1.302	0.898	0.589	0.988
Medical and health services	1	2.995	1.082	4.023	1.047
Transport	1	0.950	0.772	0.953	1.023
Communication	1	1.128	0.722	0.816	0.641
Recreation	1	1.231	0.788	0.663	1.121
Education	1	3.412	1.023	1.298	1.602
Restaurants	1	1.025	0.797	0.802	1.310
Other goods and services	1	1.632	1.000	1.000	1.215
$b(p)$	1	1.054	0.992	1.053	0.930

Notes. Listed are the reference prices used in the calculations for Table 1 and Appendix Table A1. As all prices have been normalised by the sample mean (over 124 countries), the prices $p = 1$ represent the sample mean. The second set of prices are for the US. The third set of reference prices are the GK prices computed as in (1)–(2), the fourth set are the GAIA reference prices computed as in (9)–(10), and the final set of prices are for South Korea. The final row is computed as $b(p) = \prod_i p_i^{\beta_i}$ using the β estimates from Appendix Table A2. Source. Authors’ calculations using 2005 ICP benchmark program data.

function at two different prices, but constant utility, so it can be viewed as an exact cost-of-living index in the spirit of the Allen index, while at the same time, it preserves some of the desirable aggregation properties of the Geary–Khamis system.¹⁹ For this reason, Neary (2004) refers to (9)–(10) as the Geary–Allen International Accounts (GAIA).

We have computed the GAIA reference prices π^{GAIA} for the 124 countries and 12 consumption goods using Neary’s software. We follow his procedure of first normalising the prices of each good by the arithmetic mean of the country prices, so that $\pi = 1$ is the sample mean of prices. In Table 3, we report this sample mean along with the actual US prices, the Geary–Khamis reference prices π^{GK} , the GAIA reference prices π^{GAIA} and the actual prices for South Korea. It can be seen that the GK and GAIA reference prices fall in between those of the US and South Korea for most commodities.

From (8), it is evident that, with AIDS preferences, real income at any reference prices π^B can be computed from real income at any other reference prices π^A by:

$$\frac{e(\pi^B, u_j)}{e(\pi^B, u_k)} = \left[\frac{e(\pi^A, u_j)}{e(\pi^A, u_k)} \right]^{b(\pi^B)/b(\pi^A)}, \tag{11}$$

so it is very easy to make the transformation from one reference price vector to another, as noted by Feenstra *et al.* (2009b). At the bottom of Table 3, we report the values of $b(\pi^{GK}) = 0.992$ and $b(\pi^{GAIA}) = 1.053$, which can be compared to $b(1) = 1$ at the sample mean. With these values, we can easily make the transformation from real consumption based on the GK reference prices in column (6) of Tables 1 and 2, to

¹⁹ Neary follows Geary (1958) in using the reciprocal of (10), which he calls a ‘real exchange rate’. We instead follow the PWT convention of using the purchasing power parity, defined as expenditure at domestic prices relative to expenditure at reference prices.

real consumption based on the GAIA reference prices reported in column (7). Using either the GK or GAIA reference prices, we see in columns (6) and (7) of Table 2 that real consumption per capita in China is increased by 24% due to the adjustment in prices under model 1, which is more than what we found for the consumption indexes in column (2)–(5). Under model 4, the upwards adjustment in real consumption is 22%, similar to what we found for the fixed-weight consumption indexes.

A final reference-price calculation we make is due to a suggestion by Barnett *et al.* (2009). While accepting the advantages of basing international comparisons on the expenditure function, they point out that every country would prefer to use its own prices for such comparisons. Taking a ‘democratic’ average of such aspirations, they recommend that the real incomes of any two countries j and k should be compared using *every* country’s price vector p_l , $l = 1, \dots, C$, with the preferred index set equal to the geometric mean of the resulting C comparisons. From (8), applying this procedure in the AIDS case results in:

$$\left[\prod_{l=1}^C \frac{e(p_l, u_j)}{e(p_l, u_k)} \right]^{1/C} = \left[\prod_{l=1}^C \left(\frac{u_j}{u_k} \right)^{b(p_l)} \right]^{1/C} = \left(\frac{u_j}{u_k} \right)^{\sum_{l=1}^C b(p_l)/C}. \quad (12)$$

Thus, it is apparent that in the AIDS case, this recommendation corresponds to the use of reference prices π^D where

$$b(\pi^D) = \frac{1}{C} \sum_{l=1}^C b(p_l);$$

we refer to these as ‘Diewert reference prices’ for brevity. For our sample of 124 countries, we obtain $b(\pi^D) = 0.988$, and it turns out that $b(\pi^D) \approx [b(\pi_{us})b(\pi_{kor})]^{0.5}$, which means that the Diewert reference prices are equivalent to using the geometric mean of the US and South Korean prices. With these values for $b(\pi^D)$, column (8) of Tables 1 and 2 is readily computed. Real consumption in China is revised upwards by fully 28% due to the adjustment in its prices under model 1. Using these prices, we find that real consumption per capita in China is 6.9% of that in the US, which is the highest of any consumption estimate shown in Table 1. Under model 4, we find that real consumption in China is 6.6% that of the US, as compared with 6.8% under the uniform 20% reduction in Chinese prices used by Deaton and Heston (2010) and PWT 7.0.

We conclude that the downward bias in real consumption from the ICP’s use of urban prices for China is probably quite substantial: roughly 15%–25% under model 1, depending on whether we look at the consumption indexes or the expenditure function, and about 22% in either case under model 4. These biases justify the uniform 20% reduction in consumption prices used in PWT 7.0, and which will also be used in PWT 8.0 (Feenstra *et al.*, 2013).²⁰ The question we address next is how these higher estimates for real consumption influence the total measure of real GDP.

²⁰ More precisely, PWT 7.0 and 7.1 reports two sets of estimates for China, labelled as ‘China version 1’, which uses the official ICP prices in China, and ‘China version 2’ which lowers all the prices for Chinese consumption goods by 20%. Likewise, PWT 8.0 will report these two estimates.

2. Real GDP on the Expenditure Side

The PWT defines real GDP by using the fixed-weight index in (3) to convert nominal exports and imports, or nominal GDP, to real GDP measured across countries in dollars:

$$\begin{aligned} \text{RGDP}_j^e &\equiv \frac{\text{GDP}_j}{\text{PPP}_j^e} \\ &= \sum_{i=1}^M \pi_i^e q_{ij} + (X_j - M_j)/\text{PPP}_j^e, \end{aligned} \quad (13)$$

where the equality follows from $\text{GDP}_j = \sum_{i=1}^M p_{ij} q_{ij} + (X_j - M_j)$, where X_j and M_j are the nominal values of exports and imports. Note that q_{ij} and PPP_j^e are defined as in (2) and (3) but now computed over all final goods, that is, for consumption, investment and government expenditures. We use the superscript e on real GDP^e to stress that this is an expenditure-based measure, as the prices used to compute PPP_j^e are those for final goods only. As discussed by Feenstra *et al.* (2009a), this measure of real GDP is intended to reflect the *living standards* or *consumption possibilities* of an economy. In the next section, we will discuss an alternative output-based measure, real GDP^o, that reflects the *production possibilities* of an economy.

Feenstra *et al.* (2013) compute (13) for all countries included in the 2005 ICP, as will be reported in PWT 8.0. From preliminary results for China, *without* making any adjustment to the ICP prices, they find that real GDP^e per capita is \$4,749 in 2005, or about 15% larger than the estimate of \$4,088 from the World Bank (2008a). The difference between these two estimates can only come from one of two sources:

- (i) the use of the GEKS method by the ICP/World Bank, rather than the GK method used here;
- (ii) the fact that the ICP/World Bank does not compute the real GDPs over all countries simultaneously, but rather, uses certain ‘link’ countries across regions and then computes regional and intraregional real GDP based on the linking methodology described by Diewert (2010b).²¹

Heston (2007) and Deaton and Heston (2010) argue that this linking method probably leads to an understatement of Chinese real GDP, and we concur. Interestingly, however, the understatement of real GDP for India is not quite as large: PWT 8.0 reports per capita real GDP for India as \$2,430 in 2005, which is only about 10% larger than the estimate of \$2,222 from the World Bank (2008a).

We also compute real GDP^e using the same 20% uniform reduction in Chinese prices as used in PWT 7.0. This adjustment to the final consumption goods is combined with *unadjusted* prices for investment and government expenditures, as these were not subject to the same urban bias in their collection. Using the adjusted prices for final consumption goods, PWT 8.0 finds that real GDP^e per capita in China is \$5,349, or a further 15% higher than the World Bank (2008a) estimate of \$4,088. We

²¹ PWT version 7.0 assesses the first of these reasons by providing another measure of real GDP, called ‘cgdp2’ and referred to as ‘average GEKS-CPDW’. This calculation uses the GEKS method. For ‘China version 1’, per capita real GDP is \$4,813 in 2005, and for ‘China version 2’, per capita real GDP is \$5,366 in that year. Both of these estimates exceed that obtained by PWT using the GK system. Evidently, the use of GEKS by the World Bank cannot explain its low estimate for China’s real GDP, which leaves the use of ‘link countries’ – or other unknown factors – as the culprit.

conclude that the World Bank appears to underestimate real GDP^e in China by fully 30%.

3. Real GDP on the Output Side

Feenstra *et al.* (2009a) have recently contrasted real GDP measured on the expenditure side of the economy, as done by the World Bank and PWT, with real GDP measured on the output side. To develop real GDP on the output side, suppose that the M final goods now include those used for consumption, investment and government purchases, all of which are non-traded. In addition, suppose that there are $i = M + 1, \dots, M + N$ intermediate inputs that can be both imported and exported (imports and exports are different varieties). This convention that all traded goods are by definition intermediate inputs follows the ‘production approach’ to modelling imports and exports of Diewert and Morrison (1986) and Kohli (2004), or the ‘middle products’ approach of Sanyal and Jones (1982).

Specifically, let us denote three groups of commodities:

- those for final domestic demand (quantities $q_{ij} \geq 0$ and prices $p_{ij} > 0$, for $i = 1, \dots, M$);
- those for exports (quantities $x_{ij} \geq 0$ and prices $p_{ij}^x > 0$, for $i = M + 1, \dots, M + N$);
- imported intermediate inputs (quantities $m_{ij} \geq 0$ and prices $p_{ij}^m > 0$, $i = M + 1, \dots, M + N$).

The world price vectors for exports and imports are \mathbf{p}_j^x and \mathbf{p}_j^m in country j , and domestic prices are $\mathbf{p}_j^x + \mathbf{s}_j$ and $\mathbf{p}_j^m + \mathbf{t}_j$. We use \mathbf{s}_j and \mathbf{t}_j to denote the vectors of differences between home and world prices, which may be due to export subsidies and import tariffs, respectively, or may also reflect natural trade costs. The column vector of prices is $\mathbf{P}_j = (\mathbf{p}_j, \mathbf{p}_j^x + \mathbf{s}_j, \mathbf{p}_j^m + \mathbf{t}_j)$, and we let $\mathbf{y}_j \equiv (\mathbf{q}_j, \mathbf{x}_j, -\mathbf{m}_j)$ denote the corresponding column vector of outputs and inputs. Then, the revenue or GDP function for the economy is defined as:

$$r_j(\mathbf{P}_j, \mathbf{v}_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0} \left[\mathbf{P}'_j \mathbf{y}_j | F_j(\mathbf{y}_j, \mathbf{v}_j) = 1 \right], \quad (14)$$

where $F_j(\mathbf{y}_j, \mathbf{v}_j)$ summarises the aggregate production possibilities for each country, which depend on the vector \mathbf{v}_j representing primary factor endowments in country j , and also on the subscript j representing differences in technologies across countries.

3.1. Real Output with Reference Prices

We will distinguish the reference prices π_i for final goods, $i = 1, \dots, M$, and two sets of reference prices π_i^x , π_i^m for exports and imported intermediate inputs, $i = M + 1, \dots, M + N$. Denote the $M + 2N$ dimensional vector of reference prices by $\boldsymbol{\Pi} = (\boldsymbol{\pi}, \boldsymbol{\pi}^x, \boldsymbol{\pi}^m)$. We suppose that the country is engaged in *free trade* at these reference prices, and evaluate GDP on the output side using the revenue function:

$$\text{RGDO}_j^*(\boldsymbol{\Pi}) \equiv r_j(\boldsymbol{\Pi}, \mathbf{v}_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0} \left\{ \boldsymbol{\Pi}' \mathbf{y}_j | F_j(\mathbf{y}_j, \mathbf{v}_j) = 1 \right\}. \quad (15)$$

Provided that $F_j(y_j, v_j)$ is sufficiently concave, we can expect (15) to have a well-defined solution, which we denote by $q_{ij}^*, x_{ij}^*, m_{ij}^*$. Let us make this assumption on the revenue function.

ASSUMPTION 1. For all $\Pi > 0$, $r_j(\Pi, v_j)$ is positive, bounded above and continuously differentiable.

In economic terms, the assumption that the revenue function is positive implies that even if the price of an imported intermediate input is very high, the country can economise on its imports to still produce positive revenue. The assumption that the revenue function has an upper bound means that the economy cannot make arbitrarily high revenue by importing some inputs and exporting other goods.

To obtain real GDP on the output side, we shall use the reference prices Π to obtain final demands, net outputs, exports and imports as $q_{ij}^* \equiv \partial r_j(\Pi, v_j) / \partial \pi_i$, $i = 1, \dots, M$, and, $x_{ij}^* \equiv \partial r_j(\Pi, v_j) / \partial \pi_i^x$, $-m_{ij}^* \equiv \partial r_j(\Pi^*, v_j) / \partial \pi_i^m$, $i = M + 1, \dots, M + N$. We assume that the sum across countries of each of these quantities is strictly positive.

ASSUMPTION 2. For all $\Pi > 0$, $\sum_{j=1}^J q_{ij}^* > 0$, $i = 1, \dots, M$, and $\sum_{j=1}^J m_{ij}^* > 0$, $\sum_{j=1}^J x_{ij}^* > 0$, for $i = M + 1, \dots, N$.

Notice that this assumption applies to the observed prices $P_j > 0$ and quantities too.

Consider computing the reference prices as a weighted average of free-trade prices:

$$\pi_i = \frac{\sum_{j=1}^J \left(\frac{p_{ij}}{PPP_j^*} \right) q_{ij}}{\sum_{j=1}^J q_{ij}^*}, \quad i = 1, \dots, M, \tag{16}$$

$$\pi_i^x = \frac{\sum_{j=1}^J \left(\frac{p_{ij}^x}{PPP_j^*} \right) x_{ij}}{\sum_{j=1}^J x_{ij}^*}, \quad i = M + 1, \dots, M + N, \tag{17}$$

$$\pi_i^m = \frac{\sum_{j=1}^J \left(\frac{p_{ij}^m}{PPP_j^*} \right) m_{ij}}{\sum_{j=1}^J m_{ij}^*}, \quad i = M + 1, \dots, M + N, \tag{18}$$

and,

$$PPP_j^* = \frac{r_j(P_j, v_j)}{r_j(\Pi, v_j)}, \quad j = 1, \dots, C. \tag{19}$$

Thus, in (16)–(19), we use the *optimal* quantities in the denominators but the *observed* quantities in the numerators. In (19), the purchasing power parity on the output side, PPP_j^* , is computed by comparing nominal GDP to real GDP at free-trade reference prices. The system defined in (16)–(19) is an extension of the GAIA system in Neary

(2004) by introducing exports and imports explicitly into the system. To be able to use this system, we need to demonstrate the existence of a positive solution for π_i , π_i^x , π_i^m and PPP_j^* . We follow Neary (2004) in proving the following result.

PROPOSITION 1. *Under Assumptions 1 and 2, there exists a positive solution for π_i , π_i^x , π_i^m and PPP_j^* satisfying the system (16)–(19).*

Proof. See Appendix C.

Using the reference prices coming from Proposition 1, or any other, we can make comparisons across countries of real GDP on the output side – or *real output* for short – using the ratio of revenue functions:

$$\frac{r_j(\Pi, \mathbf{v}_j)}{r_k(\Pi, \mathbf{v}_k)}. \quad (20)$$

We first show how this can be implemented using a fixed-weight index – by analogy with the expenditure function approach earlier, this is exact only if the revenue function is a Leontief function of prices – and then discuss in Section 3.3 the implications of estimating the revenue function directly.

3.2. Fixed-weight Index on the Output side

The measure of real GDP on the output side, or real GDP^o, is defined by Feenstra *et al.* (2009a) using reference prices for final outputs π_i^o , exports π_i^x and imports π_i^m , as:

$$\begin{aligned} \text{RGDP}_j^o &\equiv \sum_{i=1}^M \pi_i^o q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij}) \\ &= \sum_{i=1}^M \pi_i^o q_{ij} + \left(\frac{X_j}{PPP_j^x} \right) - \left(\frac{M_j}{PPP_j^m} \right) \end{aligned} \quad (21)$$

where X_j and M_j are the nominal values of exports and imports as in (13). The equality in the second line of (21) follows by defining the PPPs of exports and imports, over the traded goods $i = M + 1, \dots, M + N$:

$$PPP_j^x = \frac{\sum_{i=M+1}^{M+N} p_{ij}^x x_{ij}}{\sum_{i=M+1}^{M+N} \pi_i^x x_{ij}}$$

and

$$PPP_j^m = \frac{\sum_{i=M+1}^{M+N} p_{ij}^m m_{ij}}{\sum_{i=M+1}^{M+N} \pi_i^m m_{ij}}. \quad (22)$$

The measurement of real GDP^o requires disaggregate prices for traded goods, p_{ij}^x and p_{ij}^m , which are used to obtain the reference prices as a weighted average of observed prices:

$$\pi_i^o = \frac{\sum_{j=1}^C \left(\frac{p_{ij}}{PPP_j^o} \right) q_{ij}}{\sum_{j=1}^C q_{ij}}, \quad i = 1, \dots, M, \tag{23}$$

$$\pi_i^x = \frac{\sum_{j=1}^C \left(\frac{p_{ij}^x}{PPP_j^o} \right) x_{ij}}{\sum_{j=1}^C x_{ij}}, \quad i = M + 1, \dots, M + N, \tag{24}$$

$$\pi_i^m = \frac{\sum_{j=1}^C \left(\frac{p_{ij}^m}{PPP_j^o} \right) m_{ij}}{\sum_{j=1}^C m_{ij}}, \quad i = M + 1, \dots, M + N, \tag{25}$$

and,

$$PPP_j^o = \frac{GDP_j}{\sum_{i=1}^M \pi_i^o q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij})}, \quad j = 1, \dots, C, \tag{26}$$

recalling that GDP_j represents nominal GDP of country j evaluated at its own prices. As in the GK system (1)–(2), one normalisation is needed in the system (21)–(26). This system extends the GK system by adding information on export and import prices and quantities.

We follow Feenstra *et al.* (2009a) in rewriting $RGDP_j^o$ to give a clear interpretation of the difference between it and $RGDP_j^e$. Notice that $RGDP_j^o$ in (21) can be decomposed as:

$$RGDP_j^o = \left(\frac{\sum_{i=1}^M \pi_i^o q_{ij}}{\sum_{i=1}^M p_{ij} q_{ij}} \right) \sum_{i=1}^M p_{ij} q_{ij} + \left(\frac{\sum_{i=M+1}^{M+N} \pi_i^x x_{ij}}{\sum_{i=M+1}^{M+N} p_{ij}^x x_{ij}} \right) X_j - \left(\frac{\sum_{i=M+1}^{M+N} \pi_i^m m_{ij}}{\sum_{i=M+1}^{M+N} p_{ij}^m m_{ij}} \right) M_j. \tag{27}$$

We can define the three ratios appearing in (27) as the inverse of the PPPs for final expenditure, exports and imports, the latter two already given in (22) and the former by:

$$PPP_j^q \equiv \left(\frac{\sum_{i=1}^M p_{ij} q_{ij}}{\sum_{i=1}^M \pi_i^o q_{ij}} \right).$$

It will be convenient to work instead with the associated price levels for final goods, exports and imports, obtained by dividing the PPPs by the nominal exchange rate E_j :

$$PL_j^e \equiv \frac{PPP_j^e}{E_j}, \quad PL_j^q \equiv \frac{PPP_j^q}{E_j}, \quad PL_j^x \equiv \frac{PPP_j^x}{E_j}, \quad PL_j^m \equiv \frac{PPP_j^m}{E_j}.$$

Comparing (13) and (27), it follows immediately that the difference between $RGDP_j^e$ and $RGDP_j^o$ is:

$$\frac{RGDP_j^e - RGDP_j^o}{RGDP_j^e} = \left(1 - \frac{PL_j^e}{PL_j^q}\right) \left(\frac{\sum_{i=1}^M p_{ij} q_{ij}}{GDP_j}\right) + \left(1 - \frac{PL_j^e}{PL_j^x}\right) \left(\frac{X_j}{GDP_j}\right) - \left(1 - \frac{PL_j^e}{PL_j^m}\right) \left(\frac{M_j}{GDP_j}\right). \quad (28)$$

We find in practice that PL_j^e and PL_j^q are quite similar, as they are both computed from final expenditures, but with different reference prices. If these two deflators for final expenditure are equal, then either $PL_j^x > PL_j^e$ or $PL_j^m < PL_j^e$ is needed to have real GDP_j^e exceed real GDP_j^o and both inequalities holding is sufficient for this. To interpret these conditions, having export prices above their reference level and import prices below their reference level will contribute towards $RGDP_j^e$ exceeding $RGDP_j^o$. For example, proximity to markets that allow for higher export prices would work in this direction but being distant from markets leading to high import prices would work in the opposite direction, raising PL_j^m and tending to make $RGDP_j^e$ less than $RGDP_j^o$.

Empirical implementation of the fixed-weight index from the output side described in (23)–(26) is currently being undertaken as a part of the ‘next generation’ PWT 8.0 methodology described in Feenstra *et al.* (2013), using the 2005 ICP benchmark and the normalisation that $RGDP_j^e$ equals $RGDP_j^o$ for the US. It turns out that $RGDP_j^e$ is then approximately equal to $RGDP_j^o$ when summed across the 2005 sample of countries. The prices for exports and imports used in the calculations correct for the problem of mismeasured quality that are inherent in unit-value indexes, using the methods of Feenstra and Romalis (2012). It is found that per capita $RGDP_j$ for China is slightly less than $RGDP_j^e$, so China is slightly smaller when viewed from the output side. To see where this calculation comes from, we compute (28) as:

$$\frac{RGDP_j^e - RGDP_j^o}{RGDP_j^e} = [0]0.95 + \left[1 - \left(\frac{0.32}{0.67}\right)\right]0.36 - \left[1 - \left(\frac{0.32}{0.72}\right)\right]0.31 \approx 0.016.$$

The first term appearing on the right of (28) is zero, because the price levels PL_j^e and PL_j^q , both computed over final goods only, are equal at 0.32. The price level for Chinese exports is 0.67, while the price level for imports is 0.72. That means the *terms of trade* for China is $0.67/0.72 = 0.93$.²² While having a terms of trade less than unity would tend to make $RGDP_j^e$ less than $RGDP_j^o$, this tendency is offset by having nominal exports that are larger than nominal imports, which tends to make $RGDP_j^e$ exceed $RGDP_j^o$. The positive impact of China’s trade surplus exceeds the negative impact of its low terms of trade, with the net result that real GDP on the output side for China is lower than that on the expenditure side, by about 1.6%.

²² If the prices for exports and imports are not adjusted for quality, then the terms of trade for China are much lower than this amount. That explains the larger magnitude for China’s $RGDP^o$ reported in our working paper (Feenstra *et al.*, 2012).

3.3. Revenue Function Approach

The fixed-weight estimates in the previous subsection do not take account of substitution bias on the production side. The ideal correction for this would be to estimate the revenue function in (14) across a comprehensive set of countries in the same way that Neary (2004) estimates the expenditure function.²³ Unfortunately, this approach does not appear to be feasible as yet. The reason for this is that (14) depends on a full set of factor endowments v_j that differ across countries, and also depends on the subscript j representing differences in technologies across countries. Both of these features represent formidable hurdles to estimation. But, there is one theoretical result we can provide, which suggests that real GDP in China would be *even larger* if measured using the revenue function approach.

To obtain this result, let $\Pi = (\pi, \pi^x, \pi^m)$ denote the reference prices used to compute $RGDP_j^o(\Pi)$ in (21). Then, because the quantities in (21) are feasible to produce, but not optimal at the prices Π , it follows that:

$$RGDP_j^o(\Pi) \leq r_j(\Pi, v_j). \quad (29)$$

Using this inequality, we can obtain a result on the comparison of real GDP on the output side across countries. Consider reference prices that equal the observed prices of a high-income country j , $\Pi = \mathbf{P}_j = (\mathbf{p}_j, \mathbf{p}_j^x, \mathbf{p}_j^m)$. Then, it follows immediately that $RGDP_j^o = r_j(\mathbf{P}_j, v_j)$, while $RGDP_k^o(\mathbf{P}_j) \leq r_k(\mathbf{P}_j, v_k)$ for all other countries k . It follows that,

$$\frac{RGDP_k^o(\mathbf{P}_j)}{RGDP_j^o(\mathbf{P}_j)} \leq \frac{r_k(\mathbf{P}_j, v_k)}{r_j(\mathbf{P}_j, v_j)}. \quad (30)$$

That is, with reference prices of a high-income country j , the ratio of output-side real GDP for any lower-income country k relative to country j is even greater using the revenue function than in the fixed-weight GK approach. Recalling from our discussion of Table 3 that reference prices tend to be closer to the prices of countries in the upper part of the world distribution, this would imply higher levels of real GDP for poorer countries. In particular, this suggests that real GDP in China relative to the US could be even higher using the revenue function, if and when this approach proves feasible to implement.

4. Conclusions

In this study, we analyse the revision to real GDP for China made by the World Bank using prices from the 2005 round of the International Comparison Programme. Because those prices were higher than expected for China, the corresponding estimate of real GDP in China was lowered: from \$6,757 per capita in World Bank (2007) to \$4,088 in World Bank (2008*a*). Possible reasons for this downward revision have been

²³ An alternative approach, explored by Honohan (2001), would be to calibrate world production possibilities.

discussed by Deaton and Heston (2010) and here we provide a quantitative evaluation of the possibilities.

Our first objective is to examine the sensitivity of real consumption comparisons to the choice of the index number methods used. We include the fixed-weight GK index, three flexible-weight indexes – the Fisher, GEKS and CCD indexes – along with the expenditure function approaches of Neary (2004) and Barnett *et al.* (2009). In all cases, we compare estimates for real consumption in China with and without adjustments to prices to correct for the urban bias of Chinese prices in the 2005 ICP. Making use of a regression model to explain commodity-specific price levels as a function of real per capita income in other Asian countries and applying the resulting adjustments to price data for China, we measure the revisions to be anywhere between 8% and 28% depending on the index number method used. Our preferred methods give higher estimates, so we conclude that an upward revision of real consumption in China of up to 25% is quite realistic.

We then move to calculation of total real GDP, including investment, government and the trade balance. We find that the GK estimate of real GDP on the expenditure side for China is \$4,749 in 2005, which is 15% larger than the estimate from the World Bank of \$4,088. If we adjust the prices of consumption goods, then real GDP^c per capita rises to \$5,349, or another 15%. These estimates are not affected very much by a consideration of real GDP measured on the output side, from Feenstra *et al.* (2013), which is about 1.5% lower than real GDP on the expenditure side for China in 2005. Thus, our final estimate of real per capita GDP in China relative to the US in 2005 is that it is up to 30% higher than estimated by the World Bank (2008*a*).

In conclusion, our results throw light on the reasons for the unprecedented fall of 40% in the World Bank's estimate of China's 2005 GDP between 2007 and 2008. At a broader level, they also illustrate some of the difficulties in carrying out international comparisons of living standards and GDP, and point towards ways of overcoming them.

Appendix A. Consumption Data and Estimates

In Table A1, we report the calculations of real consumption for all 124 countries. In Table A2, we report the estimates of the AIDS expenditure function, listing first the R^2 values for the share equations for each product and then the parameters α , β and Γ .²⁴

²⁴ These estimates are taken from Feenstra *et al.* (2009*b*).

Table A1
Comparisons of Real Consumption and Real GDP, 2005 (US = 1)

Country	Nominal consumption per capita (1)	Consumption indexes				Using AIDS expenditure function and reference prices from:		
		Geary– Khamis (2)	Fisher (3)	GEKS (4)	CCD (5)	Geary– Khamis (6)	Neary, GAIA (7)	Diewert (8)
Iceland	1.248	0.771	0.786	0.794	0.795	0.790	0.779	0.791
Norway	1.099	0.747	0.75	0.758	0.768	0.756	0.743	0.757
Luxembourg	1.071	0.925	0.912	0.919	0.924	0.912	0.907	0.913
Switzerland	1.043	0.733	0.743	0.739	0.749	0.750	0.737	0.751
US	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Denmark	0.982	0.673	0.678	0.685	0.69	0.687	0.671	0.688
UK	0.895	0.775	0.766	0.793	0.807	0.779	0.767	0.779
Ireland	0.843	0.64	0.654	0.662	0.648	0.664	0.648	0.665
Sweden	0.838	0.681	0.695	0.693	0.696	0.694	0.679	0.695
Austria	0.814	0.761	0.753	0.779	0.791	0.768	0.756	0.769
France	0.775	0.699	0.702	0.716	0.726	0.705	0.690	0.706
Finland	0.773	0.608	0.613	0.623	0.63	0.621	0.603	0.623
Netherlands	0.756	0.706	0.702	0.72	0.736	0.698	0.682	0.699
Japan	0.743	0.633	0.646	0.618	0.625	0.659	0.642	0.660
Belgium	0.735	0.66	0.67	0.671	0.679	0.672	0.655	0.673
Canada	0.731	0.729	0.731	0.735	0.733	0.740	0.727	0.741
Germany	0.727	0.668	0.665	0.67	0.683	0.670	0.653	0.671
Australia	0.722	0.683	0.699	0.701	0.702	0.704	0.688	0.704
Italy	0.679	0.625	0.613	0.636	0.648	0.633	0.615	0.634
New Zealand	0.606	0.575	0.578	0.59	0.599	0.596	0.577	0.597
Cyprus	0.597	0.649	0.628	0.67	0.691	0.668	0.652	0.669
Spain	0.582	0.623	0.614	0.648	0.677	0.633	0.616	0.634
Greece	0.541	0.609	0.6	0.637	0.667	0.630	0.613	0.631
Hong Kong, China	0.504	0.645	0.634	0.634	0.634	0.635	0.618	0.636
Portugal	0.441	0.491	0.483	0.5	0.515	0.502	0.482	0.504
Israel	0.422	0.511	0.516	0.516	0.52	0.517	0.497	0.519
Malta	0.397	0.55	0.541	0.573	0.593	0.576	0.557	0.577
Slovenia	0.378	0.488	0.489	0.499	0.505	0.505	0.484	0.506
Singapore	0.377	0.505	0.501	0.498	0.503	0.507	0.486	0.508
Taiwan, China	0.324	0.572	0.588	0.549	0.553	0.574	0.554	0.575
Korea, Rep.	0.297	0.38	0.389	0.375	0.378	0.382	0.360	0.383
Bahrain	0.277	0.415	0.385	0.387	0.393	0.387	0.365	0.388
Czech Republic	0.238	0.433	0.431	0.429	0.436	0.433	0.411	0.434
Hungary	0.237	0.398	0.396	0.399	0.404	0.404	0.382	0.406
Macao, China	0.236	0.34	0.346	0.348	0.36	0.346	0.324	0.348
Croatia	0.226	0.347	0.34	0.347	0.353	0.351	0.329	0.352
Estonia	0.22	0.371	0.373	0.378	0.384	0.382	0.360	0.383
Poland	0.183	0.341	0.33	0.329	0.335	0.332	0.310	0.333
Lithuania	0.179	0.363	0.357	0.367	0.373	0.374	0.352	0.375
Slovakia	0.179	0.363	0.354	0.354	0.36	0.356	0.334	0.358
Mexico	0.175	0.29	0.278	0.291	0.298	0.291	0.270	0.292
Lebanon	0.169	0.289	0.28	0.277	0.283	0.286	0.265	0.287
Oman	0.153	0.249	0.229	0.237	0.24	0.238	0.218	0.239
Latvia	0.153	0.308	0.307	0.305	0.31	0.309	0.287	0.310
Chile	0.144	0.227	0.228	0.231	0.234	0.239	0.218	0.240
Mauritius	0.123	0.25	0.241	0.246	0.248	0.249	0.228	0.250
Uruguay	0.122	0.217	0.213	0.22	0.223	0.226	0.206	0.227
South Africa	0.118	0.182	0.183	0.185	0.188	0.192	0.173	0.193
Turkey	0.115	0.187	0.176	0.183	0.185	0.184	0.166	0.186
Romania	0.113	0.23	0.231	0.231	0.233	0.235	0.215	0.236

Table A1
(Continued)

Country	Nominal consumption per capita (1)	Consumption indexes				Using AIDS expenditure function and reference prices from:		
		Geary– Khamis (2)	Fisher (3)	GEKS (4)	CCD (5)	Geary– Khamis (6)	Neary, GAIA (7)	Diewert (8)
Brazil	0.101	0.176	0.176	0.177	0.18	0.186	0.168	0.187
Argentina	0.099	0.235	0.229	0.237	0.243	0.241	0.220	0.242
Bosnia and Herzegovina	0.099	0.213	0.203	0.21	0.215	0.213	0.194	0.214
Russian Federation	0.093	0.247	0.24	0.248	0.253	0.257	0.236	0.258
Fiji	0.092	0.117	0.116	0.115	0.115	0.122	0.107	0.123
Bulgaria	0.091	0.243	0.239	0.24	0.243	0.243	0.223	0.244
Serbia	0.09	0.223	0.211	0.212	0.214	0.212	0.192	0.213
Venezuela, RB	0.089	0.165	0.165	0.167	0.17	0.168	0.151	0.169
Montenegro	0.087	0.179	0.17	0.174	0.175	0.174	0.156	0.175
Malaysia	0.084	0.178	0.181	0.176	0.18	0.179	0.161	0.180
Macedonia, FYR	0.076	0.198	0.188	0.192	0.195	0.194	0.176	0.196
Jordan	0.07	0.13	0.122	0.122	0.124	0.126	0.111	0.127
Tunisia	0.068	0.141	0.131	0.14	0.144	0.140	0.124	0.141
Albania	0.068	0.14	0.138	0.14	0.142	0.141	0.125	0.142
Gabon	0.067	0.107	0.108	0.098	0.098	0.092	0.080	0.093
Kazakhstan	0.066	0.217	0.213	0.195	0.195	0.196	0.177	0.197
Namibia	0.064	0.097	0.098	0.094	0.095	0.095	0.082	0.095
Peru	0.063	0.137	0.137	0.141	0.145	0.144	0.128	0.145
Colombia	0.063	0.138	0.137	0.137	0.139	0.138	0.122	0.139
Equatorial Guinea	0.062	0.092	0.091	0.089	0.09	0.087	0.075	0.088
Ecuador	0.061	0.136	0.135	0.14	0.142	0.141	0.125	0.142
Belarus	0.061	0.24	0.222	0.22	0.222	0.219	0.199	0.220
Cape Verde	0.06	0.082	0.084	0.082	0.082	0.084	0.072	0.084
Botswana	0.057	0.102	0.104	0.098	0.096	0.096	0.083	0.097
Thailand	0.054	0.146	0.149	0.147	0.151	0.146	0.130	0.147
Swaziland	0.05	0.098	0.096	0.093	0.092	0.090	0.078	0.091
Morocco	0.044	0.073	0.071	0.073	0.075	0.075	0.064	0.076
Ukraine	0.04	0.171	0.158	0.155	0.158	0.158	0.141	0.159
Armenia	0.038	0.125	0.118	0.119	0.121	0.121	0.106	0.122
Syrian Arab Republic	0.036	0.097	0.097	0.094	0.089	0.098	0.085	0.099
Egypt, Arab Rep.	0.035	0.123	0.123	0.119	0.121	0.121	0.106	0.122
Georgia	0.033	0.109	0.104	0.103	0.103	0.103	0.089	0.104
Paraguay	0.031	0.102	0.095	0.105	0.107	0.111	0.097	0.112
Sri Lanka	0.029	0.083	0.08	0.087	0.088	0.088	0.076	0.089
Indonesia	0.028	0.073	0.072	0.073	0.074	0.075	0.064	0.076
Philippines	0.026	0.071	0.071	0.07	0.071	0.070	0.060	0.071
Moldova	0.026	0.111	0.101	0.097	0.097	0.094	0.081	0.095
Lesotho	0.026	0.06	0.058	0.055	0.054	0.053	0.045	0.054
Azerbaijan	0.025	0.097	0.094	0.094	0.095	0.095	0.082	0.096
Sudan	0.025	0.051	0.05	0.054	0.051	0.052	0.044	0.053
Bolivia	0.024	0.091	0.09	0.089	0.089	0.088	0.076	0.089
China	0.023	0.06	0.06	0.056	0.057	0.054	0.045	0.054
São Tomé and Príncipe	0.023	0.043	0.042	0.044	0.044	0.043	0.035	0.043
Cameroon	0.022	0.043	0.041	0.045	0.045	0.044	0.037	0.045
Iraq	0.021	0.068	0.064	0.06	0.059	0.059	0.050	0.060
Senegal	0.021	0.04	0.04	0.041	0.042	0.039	0.032	0.039
Nigeria	0.019	0.039	0.041	0.04	0.039	0.034	0.028	0.034
Côte d'Ivoire	0.019	0.033	0.032	0.034	0.035	0.034	0.027	0.034

Table A1
(Continued)

Country	Nominal consumption per capita (1)	Consumption indexes				Using AIDS expenditure function and reference prices from:		
		Geary– Khamis (2)	Fisher (3)	GEKS (4)	CCD (5)	Geary– Khamis (6)	Neary, GAIA (7)	Diewert (8)
Congo, Rep.	0.018	0.035	0.035	0.032	0.032	0.028	0.023	0.029
Mongolia	0.018	0.061	0.058	0.052	0.051	0.050	0.042	0.051
Angola	0.016	0.022	0.022	0.022	0.022	0.019	0.015	0.020
Kenya	0.015	0.039	0.04	0.038	0.039	0.038	0.031	0.038
Benin	0.015	0.033	0.032	0.033	0.034	0.030	0.025	0.031
Kyrgyz Republic	0.014	0.07	0.065	0.063	0.063	0.060	0.051	0.061
India	0.014	0.049	0.047	0.046	0.047	0.047	0.039	0.047
Chad	0.013	0.03	0.03	0.029	0.027	0.027	0.021	0.027
Togo	0.013	0.027	0.026	0.027	0.028	0.026	0.021	0.027
Vietnam	0.012	0.052	0.048	0.043	0.042	0.040	0.032	0.040
Cambodia	0.012	0.044	0.042	0.039	0.039	0.037	0.030	0.037
Mali	0.011	0.023	0.023	0.023	0.023	0.022	0.017	0.022
Central African Republic	0.01	0.02	0.019	0.02	0.021	0.019	0.015	0.019
Burkina Faso	0.01	0.025	0.023	0.025	0.026	0.025	0.020	0.026
Lao PDR	0.01	0.038	0.037	0.035	0.035	0.033	0.026	0.033
Nepal	0.009	0.031	0.028	0.029	0.029	0.029	0.024	0.030
Sierra Leone	0.009	0.026	0.025	0.023	0.022	0.019	0.015	0.019
Tajikistan	0.009	0.062	0.053	0.047	0.045	0.040	0.033	0.040
Madagascar	0.008	0.026	0.025	0.024	0.023	0.022	0.017	0.022
Rwanda	0.007	0.019	0.018	0.018	0.019	0.019	0.015	0.019
Guinea	0.007	0.021	0.022	0.02	0.02	0.017	0.013	0.017
Malawi	0.006	0.018	0.017	0.016	0.016	0.014	0.011	0.014
Niger	0.006	0.014	0.013	0.014	0.014	0.013	0.010	0.013
Guinea-Bissau	0.006	0.013	0.012	0.013	0.012	0.012	0.010	0.013
Liberia	0.004	0.01	0.01	0.008	0.008	0.007	0.005	0.007
Congo, Dem. Rep.	0.003	0.004	0.004	0.004	0.004	0.004	0.003	0.004
Mean	0.227	0.265	0.263	0.265	0.269	0.266	0.252	0.267
Variance	0.096	0.064	0.064	0.066	0.068	0.067	0.065	0.067
Standard error of mean	0.028	0.023	0.023	0.023	0.024	0.023	0.023	0.023

Note. Table 1 reports results for selected countries, shown here in bold.

Source. Authors' calculations as explained in the text.

Table A2
Parameter Estimates for AIDS

Category	R ²	α	β	γ											
Food and non-alcoholic beverages	0.736	0.183	-0.193	0.050	-0.035	-0.012	-0.047	0.001	-0.021	-0.023	0.013	0.034	-0.062	0.079	0.024
Alcoholic beverages and tobacco	0.172	0.033	0.004	-0.035	-0.017	0.011	-0.030	0.003	-0.016	0.037	-0.008	0.025	0.009	0.012	0.010
Clothing and footwear	0.075	0.050	-0.005	-0.012	0.011	0.009	-0.007	-0.010	-0.011	0.011	0.025	-0.029	0.016	0.020	-0.022
Gross rent, water, fuel and power	0.407	0.158	0.017	-0.047	-0.030	-0.007	0.108	0.005	-0.002	0.011	-0.003	-0.016	-0.005	0.004	-0.017
Household furnishings	0.123	0.053	0.007	0.001	0.003	-0.010	0.005	-0.027	-0.008	-0.003	0.004	0.043	0.009	0.017	-0.033
Medical and health services	0.407	0.093	0.029	-0.021	-0.016	-0.011	-0.002	-0.008	0.021	0.029	0.001	-0.007	0.008	-0.002	0.010
Transport	0.218	0.104	0.035	-0.023	0.037	0.011	0.011	-0.003	0.029	-0.041	0.001	0.051	-0.066	-0.047	0.041
Communication	0.167	0.027	0.008	0.013	-0.008	0.025	-0.003	0.004	0.001	0.001	-0.009	0.006	-0.001	-0.006	-0.023
Recreation	0.641	0.070	0.044	0.034	0.025	-0.029	-0.016	0.043	-0.007	0.051	0.006	-0.061	0.027	-0.025	-0.046
Education	0.136	0.078	-0.001	-0.062	0.009	0.016	-0.005	0.009	0.008	-0.066	-0.001	0.027	0.005	0.025	0.035
Restaurants	0.325	0.057	0.017	0.079	0.012	0.020	0.004	0.017	-0.002	-0.047	-0.006	-0.025	-0.025	-0.111	0.035
Other goods and services	0.507	0.093	0.038	0.024	0.010	-0.022	-0.017	-0.033	0.010	0.041	-0.023	-0.046	0.035	0.035	-0.014

Notes. This Table presents the AIDS estimation results for parameters α , β and γ in (7). R² gives the goodness of fit for the estimated virtual budget share of each commodity across all countries.

Appendix B. Regression Methodology for Chinese Price Adjustments

We estimate separate regressions for the following nine commodity groups within consumption: food and non-alcoholic beverages; clothing and footwear; gross rent, water, fuel and power; medical and health services; transport; recreation; education; restaurants; other goods and services. Three other commodity groups had incomplete data and so no adjustment was made for: alcoholic beverages and tobacco; household furnishings; and communication. The sample consisted of 22 developing countries in the Asia-Pacific region. Country coverage and the procedures adopted by the Asian Development Bank are detailed in their final report (Asian Development Bank, 2007). China is not included in any of the regressions since its inclusion may result in biased regression coefficients if the Chinese prices are understated in the first place.

Two regression specifications were used: linear and log-linear models:

$$\text{Linear : } Y_{ij} = \alpha + \beta X_{ij} + u_{ij}, \quad \text{and Log-linear : } \ln Y_i = \alpha + \beta \ln X_i + u_i,$$

where Y_i is the price level of the i -th commodity group in country j , defined as the ratio of the PPP for the i -th commodity group in country j to the market exchange rate for the currency of that country. Separate regressions were run for each commodity group i . We consider four alternative regressor variables leading to four different model specifications, as described in the text. In addition, we estimate two sets of models, one including Fiji and another without Fiji. As Fiji is an island country where most consumption items are imported, the price level in Fiji is considerably higher than we would expect from countries at a similar level of development. Further, Fiji price data suffered from a strong urban bias in the collection of ICP prices.

Taking account of all the combinations of alternative X variables, linear *versus* log-linear models and inclusion/exclusion of Fiji, we estimate a total of 16 alternative specifications.²⁵ To be able to compare the performance of linear models against log-linear models, we do not use the conventional correlation coefficient but rather an alternative R^2 measure, defined as the squared correlation between observed and predicted Y values. Based on R^2 , the log-linear specification with a dummy variable for Fiji dominates the linear specification in all cases, so we only report results for the former. Estimated coefficients and the R^2 values for models 1–4 estimated for the nine commodity groups are presented for the log-linear model in Table B1. Our preferred models conceptually are models 1 and 4 followed by models 2 and 3. The models offer satisfactory fits for most of the commodity groups except for transport. Table B2 shows the actual and estimated price indexes for different commodity groups derived using models 1–4.

²⁵ Detailed results for all 16 models are available from the authors.

Table B1
Estimated Regression Coefficients and Goodness of Fit

Model	X	Fiji	Constant	R ²
Food and non-alcoholic beverages				
1	0.418	0.307	2.626	0.697
2	0.204	0.497	3.697	0.830
3	0.176	0.363	3.837	0.892
4	0.245	0.408	3.476	0.824
Clothing and footwear				
1	0.132	0.334	3.867	0.293
2	0.189	0.359	3.577	0.585
3	0.166	0.232	3.691	0.649
4	0.209	0.279	3.469	0.443
Gross rent, water, fuel and power				
1	0.305	1.144	3.223	0.410
2	0.305	1.272	3.213	0.494
3	0.282	1.062	3.327	0.561
4	0.345	1.142	2.997	0.469
Medical and health services				
1	0.295	0.664	3.447	0.441
2	0.375	0.760	2.990	0.742
3	0.327	0.510	3.225	0.806
4	0.457	0.595	2.555	0.736
Transport				
1	0.007	0.457	4.696	0.153
2	0.060	0.473	4.428	0.266
3	0.068	0.426	4.387	0.333
4	0.056	0.449	4.445	0.235
Recreation				
1	0.090	0.550	4.201	0.738
2	0.143	0.634	3.920	0.784
3	0.126	0.538	4.005	0.833
4	0.159	0.573	3.834	0.702
Education				
1	0.536	0.808	1.748	0.506
2	0.515	0.933	1.920	0.802
3	0.437	0.597	2.307	0.864
4	0.636	0.706	1.277	0.866
Restaurants				
1	0.046	0.749	4.380	0.544
2	0.141	0.786	3.886	0.725
3	0.124	0.692	3.967	0.769
4	0.156	0.727	3.804	0.664
Other goods and services				
1	0.191	0.354	3.632	0.699
2	0.196	0.523	3.579	0.742
3	0.172	0.393	3.701	0.809
4	0.229	0.439	3.406	0.688

Notes. All models are in log-linear form, and the column labelled *X* shows the coefficient of the regressor. The regressor variable *X* is Model 1: the commodity-specific real expenditure index; Model 2: real GDP per capita in PPP terms; Model 3: nominal GDP per capita derived using exchange rates; Model 4: real GDP per capita consumption expenditure in PPP terms.

Table B2
Actual and Predicted Prices for China

Commodity group:	Actual price index	Predicted price indexes			
		Model 1	Model 2	Model 3	Model 4
Food and non-alcoholic beverage	112	84.1	105.9	107.2	95.3
Clothing and footwear	149	83.7	87.6	88.5	80.5
Gross rent, water charges, fuel and power	106	98.2	105.4	106.8	91.4
Medical and health services	88	122.3	117.2	119.7	96.3
Transport	102	112.8	111.4	111.3	109.1
Recreation	89	101.9	99.4	100.1	93.1
Education	115	67.1	78.0	80.6	59.2
Restaurant	114	98.1	94.9	95.6	89.0
Other goods and services	107	92.5	90.8	91.8	82.5

Appendix C. Proof of Proposition 1

We first derive a reduced system either in the international prices vector, Π , or in the output-based real exchange rates, RO . Following Neary (2004), we start with an international price vector Π^0 , which is positive. Then using Π^0 in (19), we get a value for RO_j for $j = 1, 2, \dots, C$. Substituting these RO_j s into (16), (17) and (18) yields a new price vector Π^1 . Thus, we have a function that maps Π^0 to Π^1 . We represent this relationship by:

$$\Pi^1 = H(\Pi^0) \text{ or in general, } \Pi^t = H(\Pi^{t-1}). \quad (\text{C.1})$$

(C.1) represents a system of non-linear simultaneous equations. We can state the following general properties of H :

- (i) H is positive for all $\Pi^0 > 0$;
- (ii) H is linearly homogeneous; and
- (iii) H is a continuous function.

These properties follow from the structure of the (16)–(19) and the assumed properties of the revenue function.

Furthermore, we observe that if Π is a solution to (16)–(19), then $k\Pi$ is also a solution for any $k > 0$. Thus, any solution is unique up to a factor of proportionality. Thus, we can restrict ourselves to solutions of (C.1) up to a linear restriction $\sum_{i=1}^{M+2N} \pi_i = 1$ with $\pi_j > 0$. This basically means the mapping H in (C.1) is a mapping from the unit simplex into itself. Furthermore, H is continuous. From Brouwer's fixed point theorem, it follows that there exists a fixed point Π^* such that $\Pi^* = H(\Pi^*)$. We note here that uniqueness of the fixed point is not guaranteed by this theorem.²⁶

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²⁶ Likewise, Neary (2004) proves the existence of a unique positive solution to the Geary system but only shows the existence of a positive solution to the GAIA system.

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